COMMON PRE-BOARD EXAMINATION 2023-24

Subject: MATHEMATICS BASIC (241)

Class X

MARKING SCHEME

Time: 3 Hrs. **Max. Marks: 80**

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	c) x^4y	1
2.	d) 4	1
3.	c) 2	1
4.	d) no solution	1
5.	b) Real and distinct	1
6.	c) 22 cm	1
7.	d) $\sqrt{x^2+y^2}$	1
8.	b) 10cm	1
9.	c) $\angle B = \angle D$	1
10.	c) 140^0	1
11.	c)Tan60^0	1

12.	d)15/17	1
13.	c) $\sqrt{3}$	1
14.	b) 45°	1
15.	c) 264 cm^2	1
16.	b) 2.1cm	1
17.	b) 7.8	1
18.	d)12/13	1
19.	(d) Assertion (A) is false but Reason (R) is true.	1
20.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	$kx + 3y - (k - 3) = 0$ $12x + ky - k = 0$ For a pair of linear equations to have infinitely many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$ ½ $\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$ ½ Also, $\frac{3}{k} = \frac{k-3}{k} \Rightarrow k^2 - 6k = 0 \Rightarrow k = 0, 6.$ ½ Therefore, the value of k, that satisfies both the conditions, is $k = 6$. ½	

22.	Basic proportionality theorem says that ,If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. ----------(1/2m) In ΔABC LM || CB AM/MB = AL/LC............ (1) ------------(1/2m) In ΔACD LN || CD AN/DN = AL/LC............ (2) ------------(1/2m) From equations (1) and (2) AM/MB = AN/DN ⇒ MB/AM = DN/AN Adding 1 on both sides MB/AM + 1 = DN/AN + 1 ------------(1/2m) (MB + AM)/AM = (DN + AN)/AN AB/AM = AD/AN ⇒ AM/AB = AN/AD Hence proved. **OR** triangle ABC, DE || BC Using Thales theorem (BPT - Basic proportionality theorem) $\frac{AD}{DC} = \frac{BE}{EC}$ BE X DC= AD X EC ------------(1m) => (x + 3)(3x + 4) = x(3x + 19) ------------(1/2m) => $3x^2 + 13x + 12 = 3x^2 + 19x$ => 6x = 12 ------------(1/2m) => **x = 2**	

23.	A circle touches all the four sides of a quadrilateral ABCD whose sides are cm, BC = 9 cm and CD = 8 cm Let P, Q, R and S are points where circle touches the sides AB, BC, CD and respectively. Tangents drawn from an external point to the circle to the point of contact are equal in length. From this we get -----------(1m) AB + CD = BC + AD 6 + 8 = 9 + AD -----------(1/2m) 14 = 9 + AD AD = 14 – 9 = 5 -----------(1/2m) **Length of AD is 5 cm.**	
24.	Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$ $= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$ $= \cot^2\theta$ $= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$	½ ½ ½ ½
25.	Find the perimeter of a quadrant of a circle of radius 14 cm. **[OR]** Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm. Perimeter of quadrant = $2r + \frac{1}{4} \times 2\pi r$ ⇒ Perimeter = $2 \times 14 + \frac{1}{2} \times \frac{22}{7} \times 14$ ⇒ Perimeter = 28 + 22 =28+22 = 50 cm [OR] Area of the circle = Area of first circle + Area of second circle ⇒ $\pi R^2 = \pi (r_1)^2 + \pi (r_1)^2$ ⇒ $\pi R^2 = \pi (24)^2 + \pi (7)^2$ ⇒ $\pi R^2 = 576\pi + 49\pi$ ⇒ $\pi R^2 = 625\pi$ ⇒ $R^2 = 625$ ⇒ R = 25 Thus, diameter of the circle = 2R = 50 cm.	½ ½ 1 ½ ½ 1

	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Let the fixed charge be Rs x and per km charge be Rs y. ------(1/2m) According to the question, $x + 10y = 105$ .... (1) $x + 15y = 155$ .... (2) forming eqs-----(1m) From (1), we get $x = 105 - 10y$ .... (3) Substituting the value of x in (2), we get $105 - 10y + 15y = 155$ $5y = 50$ $y = 10$ .... (4) Putting the value of y in (3), we get $x = 105 - 10 \times 10 = 5$ Solving eq to find x and y-(1m) **Hence, fixed charge is Rs 5 and per km charge = Rs 10** **Charge for 25 km = x + 25y = 5 + 250 = Rs 255** ------(1/2m) **OR**	

	Let cost of one bat be Rs x Let cost of one ball be Rs y ATQ $4x + 1y = 2050$___________(1) $3x + 2y = 1600$___________(2) $from\ (1) 4x + 1y = 2050$ $y = 2050 - 4x$ $Substite\ \ value\ of\ y\ in\ (2)$ $3x + 2(2050 - 4x) = 1600$ $3x + 4100 - 8x = 1600$ $-5x = -2500$ $x = 500$ $Substiture\ value\ of\ x\ in\ (1)$ $4x + 1y = 2050$ $4(500) + y = 2050$ $2000 + y = 2050$ $y = 50$ Hence Cost of one bat = Rs. 500 Cost of one ball = Rs. 50	 1/2 1/2 1/2 1/2 1/2 1/2 	
27.	Let us assume that $\sqrt{3}$ be a rational number $\sqrt{3} = \frac{a}{b}$ where a and b are co-prime. squaring both the sides $(\sqrt{3})^2 = \left(\frac{a}{b}\right)^2$ $3 = \frac{a^2}{b^2} \Rightarrow a^2 = 3b^2$ a^2 is divisible by 3 so a is also divisible by 3_________(1) *let* $a = 3c$ for any integer c. $(3c)^2 = 3b^2$ $9c^2 = 3b^2$ $b^2 = 3c^2$ $since\ b^2$ is divisible by 3 so, b is also divisible by 3 _____(2) From (1) & (2) we can say that 3 in a factor of a and b which is contradicting the fact that a and b are co- prime. Thus, our assumption that $\sqrt{3}$ is a rational number is wrong. Hence, $\sqrt{3}$ is an irrational number.	 1 1/2 1/2 1/2 1/2	

28. $f(x)=x^2-2x+3$ have zeroes α,β

$\Rightarrow \alpha+\beta=2$ -----------(1/2m)

$\Rightarrow \alpha\cdot\beta=3$ ------------(1/2m)

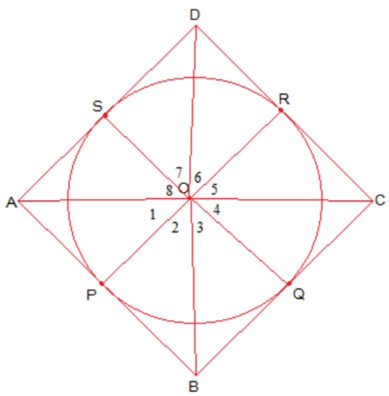
Now polynomial having $\alpha+2,\beta+2$ as roots is

$p(x)=x^2-(\alpha+2+\beta+2)x+(\alpha+2)(\beta+2)$ --------------(1m)

$=x^2-(\alpha+\beta+4)x+\alpha\beta+2(\alpha+\beta)+4$

$=x^2-(2+4)x+3+2(2)+4$

29.



--------1/2 m

Let ABCD be a quadrilateral circumscribing a circle with O such that it touches the circle at point P, Q, R, S.

Join the vertices of the quadrilateral ABCD to the center of the circle.

In ΔOAP and ΔOAS,

AP = AS (**Tangents from the same point**) --------1/2 m

OP = OS (**Radii of the circle**)

OA = OA (**Common side**)

ΔOAP ≅ ΔOAS (**SSS congruence condition**)

∴ ∠POA = ∠AOS

⇒∠1 = ∠8

Similarly we get,

∠2 = ∠3

∠4 = ∠5 --------1/2 m

∠6 = ∠7

	Adding all these angles, ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 + ∠7 +∠8 = 360° --------1/2 m ⇒ (∠1 + ∠8) + (∠2 + ∠3) + (∠4 + ∠5) + (∠6 + ∠7) = 360° ⇒ 2 ∠1 + 2 ∠2 + 2 ∠5 + 2 ∠6 = 360° --------1/2 m ⇒ 2(∠1 + ∠2) + 2(∠5 + ∠6) = 360° ⇒ (∠1 + ∠2) + (∠5 + ∠6) = 180° --------1/2 m ⇒ ∠AOB + ∠COD = 180° Similarly, we can prove that ∠ BOC + ∠ DOA = 180°	
30.	LHS = (cosecA - sinA)(secA - cosA) $=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$ -------(1/2m) $=\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$ -------(1/2m) $=\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$ -------(1/2m) $=\sin A\cos A$ $\text{RHS}=\frac{1}{\tan A+\cot A}$ $=\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$ -------(1/2m) $=\frac{\sin A\cos A}{\sin^2 A+\cos^2 A}$ -------(1/2m) $=\sin A\cos A$ -------(1/2m) L.H.S. = R.H.S. OR	

	$LHS=(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)$ $=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$ -------(1/2m) $=\left(\frac{\sin A+\cos A-1}{\sin}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$ -------(1/2m) $=\frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$ -------(1m) $=\frac{1+2\sin A\cos A-1}{\sin A\cos A}$ $\left[\because \sin^2 A+\cos^2 A=1\right]$ -------(1m) $=2.$ Hence proved	
31.	Total number of cards in a deck of cards = 52 $\therefore$Total number of outcomes = 52 $\therefore P(A)=\frac{\text{Outcomes in favour of event A}}{\text{Total number of outcomes}}$ =2/52 =1/26 ------(1m) (ii) Let B denote the event of getting a face card. $\therefore P(B)=\frac{\text{Outcomes in favour of eventB}}{\text{Total number of outcomes}}=\frac{12}{52}=\frac{3}{13}$ ------(1m) (iii) Let C denote the event of getting a queen of diamond. $\therefore P(C)=\frac{\text{Outcomes in favour of eventC}}{\text{Total number of outcomes}}=\frac{1}{52}$ ------(1m)	
	SECTION D	
	Section D consists of 4 questions of 5 marks each	

32\. Let the first tap takes x hours to completely fill tank ------(1/2m)

⇒ Second tap will take 2 hours less

⇒ According to question ------(1m)

$$\frac{1}{x}+\frac{1}{x-2}=\frac{8}{15}$$

$$\frac{x-2+x}{x(x-2)}=\frac{8}{15}$$

$$\frac{2x-2}{x(x-2)}=\frac{18}{15}$$

$$\frac{(2x-1)}{x(x-2)}=\frac{18}{15}$$

$$15(x-1)=4x(x-2)$$

$$15x-15=4x^2-8x$$

$$4x^2-23x+15=0$$ ------(1m)

$$4x^2-20x-3x+15=0$$

$$4x(x-5)-3(x-5)=0$$

$$(x-5)(4x-3)=0$$

$x=5$ or $\frac{3}{4}$ -------(1m)

Since $\frac{3}{4}-2=$ Negative time $\frac{3}{4}$ is not possible. ------(1/2m)

Which is not possible

⇒ x = 5

Rate of 1st pipe = 5 hours

Rate of 2nd pipe = 5 - 2 = 3hours ------(1m)

OR

Let the speed of train be x km /h ------(1/2m)

Distance = 180 km

So, time = 180 / x

When speed is 9 km/h more, time taken = 180 / x+9 ------(1m)

According to the given information:

180 / x - 180 / x+9 = 1

180 (x+9-x) / x(x+9) = 1 ------(1m)

180 * 9 = x(x+9)

1620 = x^2 + 9x

x^2 + 9x - 1620 = 0 ------(1m)

x^2 + 45x - 36x - 1620 = 0

x(x+45) - 36(x+45) = 0

(x-36)(x+45) = 0 ------(1m)

x = 36 or -45

But x being speed cannot be negative. ------(1/2m)

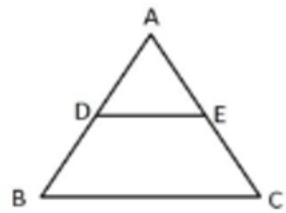
So, x = 36

Hence, the speed of the train is 36km/h.

33. **Statement**:

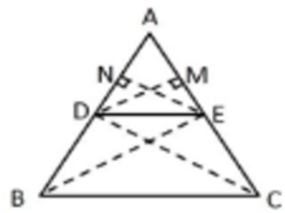
If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. -------(1/2m)

A ΔABC in which $DE \parallel BC$ and DE intersects AB and AC at D and E respectively.



To prove that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



Construction:

Join BE and CD.

Draw $EL \perp AB$ and $DM \perp AC$

---(1/2m)

.

$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EL$$ ---(1/2m)

$$ar(\Delta DBE) = \frac{1}{2} \times DB \times EL$$

---(1/2m)

Therefore the ratio of these two is $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{AD}{DB}$ . . . . (1)

---(1/2m)

Similarly,

$$ar(\Delta ADE) = ar(\Delta ADE) = \frac{1}{2} \times AE \times DM$$

$$ar(\Delta ECD) = \frac{1}{2} \times EC \times DM$$

Therefore the ratio of these two is $\frac{ar(\Delta ADE)}{ar(\Delta ECD)} = \frac{AE}{EC}$ . . . . (2)

Now, ΔDBE and ΔECD being on the same base DE and between the same parallels DE

BC, we have,

$ar(\Delta DBE) = ar(\Delta ECD)$. . . . . (3) ---(1/2m)

From equations 1, 2, 3 we can conclude that ---(1/2m)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence we can say that the basic proportionality theorem is proved.

In Δ POQ, AB || PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (basic proportionality theorem) (i) ---(1/2m)

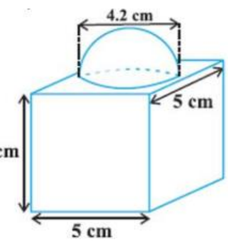
34.

Total surface area of block

= Total Surface area of cube

+ Curved Surface area of hemisphere

− Base area of hemisphere ---(1m)



Total Surface area of cube

Given that side of cube is 5 cm

Total Surface area of the cube = 6 (side)^2

= 6 (5)^2

= 6× 5 × 5 ---(1/2m)

= 150 cm^2

Curved surface area of hemisphere

Diameter of hemisphere = 4.2 cm

So, radius = r = $\frac{\text{Diameter}}{2} = \frac{4.2}{2}$ = 2.1 cm ---(1/2m)

Curved surface area of hemisphere = $2\pi r^2$

$= 2\times \frac{22}{7} \times (2.1)^2$

$= 2\times \frac{22}{7} \times 2.1 \times 2.1$

= 27.72 cm^2 ---(1m)

Base area of hemisphere

Base of hemisphere is a circle with radius 2.1 cm

Base area of hemisphere = Area of circle

$= \pi r^2$

$= \frac{22}{7} \times (2.1)^2$

$= \frac{22}{7} \times 2.1 \times 2.1$

$= 22 \times 0.3 \times 2.1$

$= 13.86 \text{ cm}^2$ ---(1m)

Therefore,

Total surface area of block

= Total Surface area of cube

+ Curved Surface area of hemisphere

– Base area of hemisphere

= (150) + (27.72) – (13.86)

= 177.72 – 13.86 ---(1m)

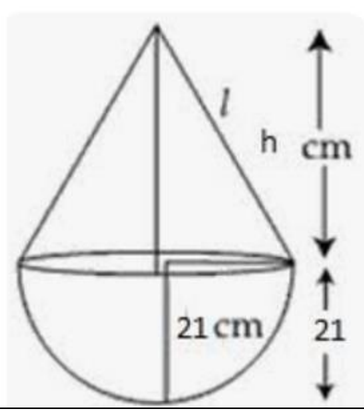
= 163.86 cm^2

OR

Let the height of the conical part be *h*.

Radius of the cone = Radius of the hemisphere = *r* = 21 cm

The toy can be diagrammatically represented as



Volume of the cone $= \frac{2}{3} \cdot$ Volume of the hemisphere Equ---(1/2m)

$$\therefore \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$$ Formula------(1 m)

$$\Rightarrow h = \frac{\frac{2}{3} \times \frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^2}$$

$$\Rightarrow h = \frac{4}{3} r$$

$$\therefore h = \frac{4}{3} \times 21cm = 28cm$$ ------------(1/2 m)

Thus, surface area of the toy = Curved surface area of cone + Curved surface area of hemi

------------(1/2 m)

Formula ----------(1 m)

= $\pi rl + 2\pi r^2$

Finding "l"---(1 m)

$$= \pi r\sqrt{h^2 + r^2} + 2\pi r^2$$

$$= \pi r\left(\sqrt{h^2 + r^2} + 2r\right)$$

$$= \frac{22}{7} \times 21cm\left(\sqrt{(28cm)^2 + (21cm)^2} + 2 \times 21cm\right)$$

$$= 66\left(\sqrt{784 + 441} + 42\right)cm^2$$

$$66\left(\sqrt{1225} + 42\right)cm^2$$

= 66(35+42) cm^2

= 66 x 77 cm^2

= 5082 cm^2 Correct answer--- (1/2m)

35.

Class interval	Frequency	Cumulative frequency
0 - 10	f1	f1
10 - 20	5	5 + f1
20 - 30	9	14 + f1
30 - 40	12	26 + f1
40 - 50	f2	26 + f1 + f2
50 - 60	3	29 + f1 + f2
60 - 70	2	31 + f1 + f2

Table with correct entries ---------(1 ½ m)

Given

Median = 32.5

The median class = 30 - 40 -----(1m)

l = 30, h = 40 - 30 = 10, f = 12 and F = 14 + f1

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$ ---(1/2m)

$$\Rightarrow 32.5 = 30 + \frac{20 - (14 + f1)}{12} \times 10$$

$$\Rightarrow 32.5 - 30 = \frac{20 - 14 - f1}{12} \times 10$$

$$\Rightarrow 2.5 = \frac{6 - f1}{12} \times 10$$

$$\Rightarrow 2.5 = \frac{6 - f1}{6} \times 5$$

⇒ 2.5 x 6 = (6 - f1) x 5

	$\Rightarrow 2.5 \times 6 = (6 - f1) \times 5$ $\Rightarrow 15 = (6 - f1) \times 5$ $\Rightarrow 15/5 = 6 - f1$ $\Rightarrow 3 = 6 - f1$ $\Rightarrow f1 = 6 - 3$ --------(1 ½ m) $\Rightarrow f1 = 3$ Given sum of frequencies = 40 $\Rightarrow f1 + 5 + 9 + 12 + f2 + 3 + 2 = 40$ $\Rightarrow 3 + 5 + 9 + 12 + f2 + 3 + 2 = 40$ -------(1/2m) $\Rightarrow 34 + f2 = 40$ $\Rightarrow f2 = 6$ $\therefore f1 = 3$ and $f2 = 6$	
	SECTION E	
36.	**Case study-1** i) AP: 10, 16, 22, ... ---------(1m) ii) $a = 10, d = 6, n = 10,$ $a_{10} = a + 9d$ $a_{10} = 10 + 9 \times 6 = 64$ --------(1m) iii) $S_8 = \frac{8}{2}(2 \times 10 + 7 \times 6)$ Formula—(1m) =248m Substituting and finding value---(1m) **OR** $S_{10} = \frac{10}{2}(10 + 64) = 370m$ Formula—(1m) Substituting and finding value---(1m)	
37.	**Case Study – 2** **i)**	

	LB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow$ LB = $\sqrt{(0-5)^2 + (7-10)^2}$ LB = $\sqrt{(5)^2 + (3)^2} \Rightarrow$ LB = $\sqrt{25+9}$ LB = $\sqrt{34}$ Hence the distance is 150 $\sqrt{34}$ km ii) Coordinate of Kota (K) is $\left(\frac{3 \text{ x } 5 + 2 \text{ x } 0}{3+2}, \frac{3 \text{ x } 7 + 2 \text{ x } 10}{3+2}\right)$ $= \left(\frac{15+0}{5}, \frac{21+20}{5}\right) = \left(3, \frac{41}{5}\right)$ iii) L(5, 10), N(2,6), P(8,6) LN = $\sqrt{(2-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ NP = $\sqrt{(8-2)^2 + (6-6)^2} = \sqrt{(4)^2 + (0)^2} = 4$ PL = $\sqrt{(8-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2} \Rightarrow$ LB = $\sqrt{9+16} = \sqrt{25} = 5$ as LN = PL ≠ NP, so Δ LNP is an isosceles triangle. **[OR]** Let A (0, b) be a point on the y – axis then AL = AP $\Rightarrow \sqrt{(5-0)^2 + (10-b)^2} = \sqrt{(8-0)^2 + (6-b)^2}$ $\Rightarrow (5)^2 + (10-b)^2 = (8)^2 + (6-b)^2$ $\Rightarrow 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$ So, the coordinate on y axis is $\left(0, \frac{25}{8}\right)$	½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½
38.	**Case study 3:**	

	(i) $\sin 60° = \frac{PC}{PA}$ --------1/2m $\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}$ m --------1/2 m **(ii)** $\sin 30° = \frac{PC}{PB}$ --------1/2m $\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36$ m --------1/2 m **(iii)** $\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC}$ $\Rightarrow AC = 6\sqrt{3}$ m --------1/2 m $\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB}$ --------1/2m $\Rightarrow CB = 18\sqrt{3}$ m ---------1/2m Width AB = AC + CB = $6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}$ m --------1/2 m **or** RB = PC =18 m & PR = CB = $18\sqrt{3}$ m --------1/2 m $\tan 30° = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}}$ --------1/2m $\Rightarrow QR = 18$ m ---------1/2m QB = QR + RB = 18 + 18 = 36m. --------1/2 m Hence height BQ is 36m	