SET	ABC

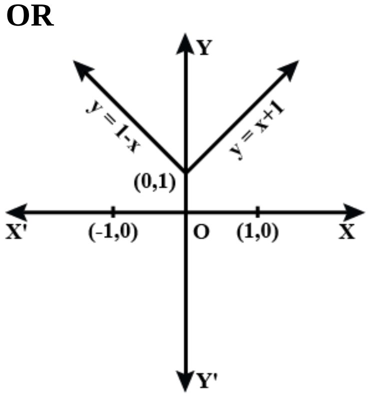
INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
MATHEMATICS (041)

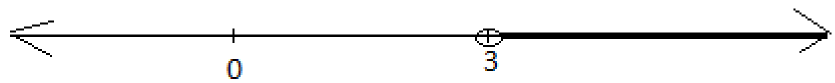
CLASS: XI Max.Marks: 80

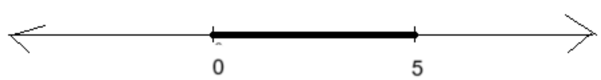
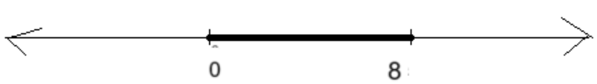
MARKING SCHEME					
SET	QN.NO	VALUE POINTS			MARKS SPLIT UP
A	1	(d)$A \cup B = \{1, 2, 3, 5, 9\}$	(b) 2	(d) $2^{mn} - 1$	1
	2	(a)$\{x : \|x\| = 5, x \in N\}$	(d) - $\frac{3}{5}$	(b) x = 4n	1
	3	(c) $\{1, 2, 3\}, \{7, 5\}$	(c) -1 + i	(a) 27 < x < 2	1
	4	(d) $2^{mn} - 1$	(c) 0	(c) y = $(x + 2)^2 + 1$	1
	5	(c) 0	(c) 2^m	(d) - $\frac{3}{5}$	1
	6	(b) 2	(c) y = $(x + 2)^2 + 1$	(c) f is a relation but not a function from A to B	1
	7	(b) x = 4n	(a) 27 < x < 2	(d)$A \cup B = \{1, 2, 3, 5, 9\}$	1
	8	(c) -1 + i	(a)$\{x : \|x\| = 5, x \in N\}$	(a) 373	1
	9	(c) y = $(x + 2)^2 + 1$	(a) 51	(c) $\frac{-1}{i+1}$	1
	10	(a) 373	(b) x = 4n	(c) $\{1, 2, 3\}, \{7, 5\}$	1
	11	(d) - $\frac{3}{5}$	(d) IV quadrant	(a) 51	1
	12	(a) 27 < x < 2	(d) $A \cup B = \{1, 2, 3, 5, 9\}$	(d) IV quadrant	1
	13	(c) 0	(c) $\frac{-1}{i+1}$	(c) 0	1
	14	(a) 51	(c) f is a relation but not a function from A to B	(c) -1 + i	1
	15	(c) 2^m	(c) $\{1, 2, 3\}, \{7, 5\}$	(c) 2^m	1

	16	(c) f is a relation but not a function from A to B	(c) 0	(a)$\{x : \|x\| = 5, x \in N\}$	1
	17	(d) IV quadrant	(a) 373	(c) 0	1
	18	(c) $\frac{-1}{i+1}$	(d) $2^{mn} - 1$	(b) 2	1
	19	(a) Both A and R are true, and R is the correct explanation of A.	(c) A is true but R is false.	(a) Both A and R are true, and R is the correct explanation of A	1
	20	(c) A is true but R is false.	(a) Both A and R are true, and R is the correct explanation of A	(c) A is true but R is false.	1
	21	Smallest set = {3, 5, 9} Largest set = {1, 2, 3, 5, 9}			1 1
	22	X = 3; y = -1			1 + 1
	23	LHS = cos 510° cos 330° + sin 390° cos 120° = cos(360° + 150°) cos(360° – 30°) + sin(360° + 30°) x cos(180° – 60°) = cos 150° cos 30° + sin 30° (-cos 60°) = cos(180° – 30°) cos 30° + sin 30° cos 60° = -cos 30° cos 30° + $\frac{1}{2} \times (\frac{-1}{2})$ $= -\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$ $= -\frac{3}{4} - \frac{1}{4}$ $= \frac{-3-1}{4}$ = - 1 **OR**			½ ½ ½ ½

		$= 2(\sin \frac{\pi}{6})^2 + (\text{cosec}(\pi + \frac{\pi}{6}))^2 (\cos\frac{\pi}{3})^2$ $= 2(\frac{1}{2})^2 + (-\text{cosec}\frac{\pi}{6})^2 (\frac{1}{2})^2$ ($\because \sin\frac{\pi}{6} = \cos\frac{\pi}{3} = \frac{1}{2}$) $= \frac{2}{4} + (-2)^2 \times \frac{1}{4}$ ($\because \text{cosec}\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}} = 2$) $= \frac{1}{2} + 4 \times \frac{1}{4} = \frac{1}{2} + 1$ $= \frac{3}{2}$	½ ½ ½ ½
	24	$= \frac{(5+\sqrt{2}i)}{(1-\sqrt{2}i)} \times \frac{(1+\sqrt{2}i)}{(1+\sqrt{2}i)}$ $= \frac{5 + 5\sqrt{2}i + \sqrt{2}i + 2(-1)}{1 - 2(-1)}$ $= \frac{5 + 5\sqrt{2}i + \sqrt{2}i - 2}{1+2}$ $= \frac{5 - 2 + 5\sqrt{2}i + \sqrt{2}i}{3}$ $= \frac{3 + 6\sqrt{2}i}{3}$ $= \frac{3(1 + 2\sqrt{2}i)}{3}$ $= 1 + 2\sqrt{2}i$	½ ½ ½ ½
	25	$\Rightarrow \frac{P(n-1,3)}{P(n,4)} = \frac{1}{9}$ $\Rightarrow \frac{(n-1)!}{(n-1-3)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$ $\Rightarrow \frac{1}{n} = \frac{1}{9}$.......As $n = n(n-1)!$ $\Rightarrow n = 9$ **OR**	½ 1 ½

		4 vowels and 4 consonants. Total 8 letters. No. of words = $4! \times 4! = 24 \times 24 = 576$ Because 4 vowels are to be in old places and the 4 consonants are to be adjusted in the remaining places.	1 + 1
	26	(i) 17 (ii) 19	1 ½ 1 ½
	27	$16 - x^2 \geq 0$ $16 \geq x^2$ Therefore, $x \leq 4$ or $x \geq -4$ The domain $[-4, 4]$ Range: f(x) is maximum at $x = 0, f(x) = 4$ And f(x) is minimum at $x = 4, f(x) = 0$ Range $[0, 4]$ **OR**  	1 ½ ½ ½ ½ finding the values 2 Graph 1
	28	$\text{LHS} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$ $= \frac{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right) + 2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right) + 2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}$ $= \frac{\sin 6x \cos x + \sin 6x \cos 3x}{\cos 6x \cos x + \cos 6x \cos 3x} = \tan 6x = \text{RHS}$	 1 ½ 1 ½

	29	As $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ and sin is positive in 2nd quadrant $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{1}{3}\right)}{2}} = \pm\sqrt{\frac{4}{6}}$ $\quad \therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}}$ $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\left(-\frac{1}{3}\right)}{2}} = \pm\sqrt{\frac{1}{3}}$ $\quad \therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$ $\therefore \tan\frac{x}{2} = -\sqrt{2}$	1 ½ 1 ½
	30	$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ and $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ $\Rightarrow \frac{10x+3x}{8} > \frac{39}{8}$ and $\frac{2x-1-4x+4}{12} < \frac{3x+1}{4}$ $\Rightarrow \frac{13x}{8} > \frac{39}{8}$ and $\frac{-2x+3}{12} < \frac{3x+1}{4}$ $\Rightarrow 13x > 39$ and $-2x+3 < 9x+3$ $\Rightarrow x > 3$ and $-11x < 0$ $\Rightarrow x > 3$ and $x > 0$ $\Rightarrow x \in (3, \infty)$ and $x \in (0, \infty) \Rightarrow x \in (3, \infty)$  	1 1 1
	31	$(1+5)^n = {}^nC_0 + {}^nC_1 5 + {}^nC_2 5^2 + {}^nC_3 5^3 + \ldots + {}^nC_n 5^n$ $6^n = 1 + n5 + 5^2({}^nC_2 + {}^nC_3 5 + \ldots + {}^nC_n 5^{n-2})$ $\therefore\ 6^n - 5n = 1 + 25k$ where $k = {}^nC_2 + {}^nC_3 5 + \cdots + {}^nC_n 5^{n-2}$ $\therefore$ 6^n - 5n leaves remainder 1 when divided by 25. **OR**	1 ½ 1 ½

		$98^5 = (100-2)^5$ $= 100^5 - {}^5C_1.100^4.2 + {}^5C_2.100^3 2^2 - {}^5C_3.100^2.2^3 + {}^5C_4.100.2^4 - {}^5C_2$ $= 100^5 - 10.100^4 + 40.100^3 - 80.100^2 + 80.100 - 32$ $= 90.100^4 + 40.100^3 - 80.100 + 8000 - 32$ $= 9000000000 + 40000000 - 300000 + 8000 - 32$ $= 9040000000 - 800000 + 7968$ $= 9039200000 + 7968$ $= 9039207968.$	1 ½ 1 ½
	32	a) The number of students who offered all three subjects is 3 b) The number of students who offered mathematics is 15+37+3+7=62 c) The number of students who did not offer any of the above three subjects is 100−(15+37+7+3+8+17+12) =1 **OR** c) The number of students who offered mathematics and statistics but not physics is 7	1 1 2
	33	(a) $25x \le 100$ (b) $x \le 4$ (c) $25x \le 125$; $x \le 5$;  **OR** $20x \le 160$; $x \le 8$  	1 1 2
	34	(i) 6! (ii) 10^6 (iii) 5! X 2! OR 5! = 120	1+1+2

	35	$\sin^2 x + \cos^2 x = 1$	½
		$\cos^2 x = 1 - \frac{9}{25}$	
		$\cos x = \frac{-4}{5}$ Since x lie in the 2nd quadrant so cosx is negative	1
		$\sin^2 y + \cos^2 y = 1$	½
		$\sin^2 y = 1 = -\cos^2 y \quad = 1 - \frac{144}{169}$	
		$\sin y = \frac{5}{13}$ Since y lie in the 2nd quadrant so siny is positive	½
		$\sin(x+y) = \frac{3}{5} \times \frac{-12}{13} + \frac{-4}{5} \times \frac{5}{13}$	1
		$= \frac{-36}{65} - \frac{20}{65}$	
		$= \frac{-56}{65}$	½
		OR	
		$= \frac{1+\cos 2x}{2} + \frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2} + \frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}$	
		$= \frac{1}{2}\left[1+\cos 2x + 1 + \cos\left(2x+\frac{2\pi}{3}\right) + 1 + \cos\left(2x-\frac{2\pi}{3}\right)\right]$	1
			1
		$= \frac{1}{2}\left[3+\cos 2x + \cos\left(2x+\frac{2\pi}{3}\right) + \cos\left(2x-\frac{2\pi}{3}\right)\right]$	½
		$= \frac{1}{2}\left[3+\cos 2x + 2\cos\left(\frac{2x+\frac{2\pi}{3}+2x-\frac{2\pi}{3}}{2}\right).\cos\left(\frac{2x+\frac{2\pi}{3}-\left(2x-\frac{2\pi}{3}\right)}{2}\right)\right]$	1
		$= \frac{1}{2}\left[3+\cos 2x + 2\cos 2x \cos\left(\pi - \frac{\pi}{3}\right)\right]$	
		$= \frac{1}{2}\left[3+\cos 2x + 2\cos 2x \left(-\cos\left(\frac{\pi}{3}\right)\right)\right]$ (As $\cos(\pi-\theta) = -\cos\theta$)	1
		$= \frac{1}{2}\left[3+\cos 2x + 2\cos 2x \left(-\frac{1}{2}\right)\right]$	
		$= \frac{1}{2}[3+\cos 2x - \cos 2x]$	
		$= \frac{1}{2}[3+0]$	½
		$= \frac{3}{2}$	

	36	$\Rightarrow (x-iy)^2 = \frac{a-ib}{c-id} \times \frac{c+id}{c+id} = \frac{(ac+bd)-i(bc-ad)}{c^2+d^2}$	1
		$\Rightarrow (x^2-y^2)-i(2xy) = \left(\frac{ac+bd}{c^2+d^2}\right) - i\left(\frac{bc-ad}{c^2+d^2}\right)$	1
		Equating real and imaginary parts on both sides, we get	
		$x^2-y^2 = \frac{ac+bd}{c^2+d^2}$ and $2xy = \frac{bc-ad}{c^2+d^2}$	½
		Now, $(x+iy)^2 = (x^2-y^2)+i(2xy) = \left(\frac{ac+bd}{c^2+d^2}\right) + i\left(\frac{bc-ad}{c^2+d^2}\right)$	1
		$\Rightarrow (x+iy)^2 = \frac{(ac+bd)+i(bc-ad)}{c^2+d^2} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \frac{a+ib}{c+id}$	
		$\Rightarrow x+iy = \sqrt{\frac{a+ib}{c+id}}$	
		$\text{LHS} = (x^2+y^2)^2 = [(x-iy)(x+iy)]^2 = (x-iy)^2(x+iy)^2$	1
		$= \left(\frac{a-ib}{c-id}\right)\left(\frac{a+ib}{c+id}\right)$	½
		$= \frac{a^2+b^2}{c^2+d^2}$	
		$= \text{RHS}$	

	37	(i) Total number of ways = ${}^{4}C_3 \times {}^{9}C_4$	1
		$= \frac{4!}{3!(4-3)!} \times \frac{9!}{4!(9-4)!}$ $= \frac{4!}{3!1!} \times \frac{9!}{4!(5)!}$ $= \frac{9!}{3!(5)!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{(3 \times 2 \times 1) \times (5)!} = \mathbf{504}$	2
		(ii) atleast 3 girls This means that there can be either 3 or 4 girls in the committee of 7. • No. of ways of to selecting 3 girls from 4 and 4 boys from 9 to form a committee of 7 = ${}^{4}C_3 \times {}^{9}C_4 = 4 \times 126 = 504$ • No. of ways of to selecting 4 girls from 4 and 3 boys from 9 to form a committee of 7 = ${}^{4}C_4 \times {}^{9}C_3 = 1 \times 84 = 84$ The total no. of ways = 504 + 84 = 588.	
		(iii) atmost 3 girls This means that there can be 0 or 1 or 2 or 3 girls in the committee of 7. The total no. of ways = 36 + 336 + 756 + 504 = 1632	2
		OR The alphabetical order of the letters of the word RACHIT is: A, C, H, I, R, T. Number of words beginning with A = 5! Number of words beginning with C = 5! Number of words beginning with H = 5! Number of words beginning with 1 = 5!	1 x 4 = 4
		Clearly, the first word beginning with R is RACHIT. ∴ Rank of the word RACHIT in dictionary = 4×5! + 1= 4 x 120+1= 481.	1
	38	Using Binomial theorem, $(x+1)^6 = {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6$	1
		$(x-1)^6 = {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$	1
		$\therefore (x+1)^6 + (x-1)^6 = 2[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6] = 2[x^6 + 15x^4 + 15x^2 + 1]$	1
		By putting $x = \sqrt{2}$ we get, $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$	1
		$= 2(8 + 15 \times 4 + 15 \times 2 + 1)$ $= 2(8 + 60 + 30 + 1)$	½
		$= 2(99)$ $= 198$	½

Set B	22	X = 1 and y = 3	2
	23	X = Y they are equal sets	2
	28	$(-\infty, -2)$	Same like set A
	30	$LHS = \sin 3x + \sin 2x - \sin x$ $= 2\sin\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right) - 2\sin\frac{x}{2}\cos\frac{x}{2}$ $= 2\cos\frac{x}{2}\left(\sin\frac{5x}{2} - \sin\frac{x}{2}\right) = 2\cos\frac{x}{2}\cos\frac{3x}{2}\sin x$ $= 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2} = RHS$	1½ 1½
	31	Domain [-3, 3] Range [0, 3]	Same like set A
	35	C (52,4) = 2,70,725 (i) 2860 (ii) 13^4 (iii) 495 (iv) 105625 (v) 29900	1 ½ 1 1 ½ 1
	OR	49^{TH} word is NAAGI. 50^{TH} word is NAAIG	Same like set A
	36	Using Binomial theorem, $(x+1)^6 = {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6$ $(x-1)^6 = {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$ $\therefore (x+1)^6 + (x-1)^6 = 2[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6] = 2[x^6 + 15x^4 + 15x^2 + 1]$ By putting $x = \sqrt{3}$ we get, final answer as 416	3 2
C		Refer answers both in Set A and Set B	