COMMON PRE-BOARD EXAMINATION 2023-24
Subject: MATHEMATICS (041)
Class XII

Time: 3 Hours Max Marks: 80

General Instructions:
1. This question paper is divided in to 5 sections - A, B, C, D and E
2. Section A comprises of 20 MCQ type questions of 1 mark each.
3. Section B comprises of 5 Very Short Answer Type Questions of 2 marks each.
4. Section C comprises of 6 Short Answer Type Questions of 3 marks each.
5. Section D comprises of 4 Long Answer Type Questions of 5 marks each.
6. Section E comprises of 3 source based / case based / passage-based questions (4 marks each) with sub parts.
7. Internal choice has been provided for certain questions

SECTION – A
(Each MCQ Carries 1 Mark)

1 If A = $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to

a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

2 The value of '*k*' for which the function *f(x)* = $\begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x = 0 is

a) 0 b) -1 c) 1 d) 2

3 Find k, if A = $\begin{bmatrix} -2 & 3 \\ k & 4 \end{bmatrix}$ is a singular matrix

a) $\frac{3}{8}$ b) $\frac{-3}{8}$ c) $\frac{-8}{3}$ d) $\frac{8}{3}$

4 The value of $\int_2^3 \frac{x}{x^2+1} dx$ is

a) log 4 b) log $\frac{3}{2}$ c) $\frac{1}{2}$ log 2 d) log $\frac{9}{4}$

5 The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is

a) 2 b) 3 c) 4 d) 5

6 The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If z = 18x + 9y be the objective function, then:

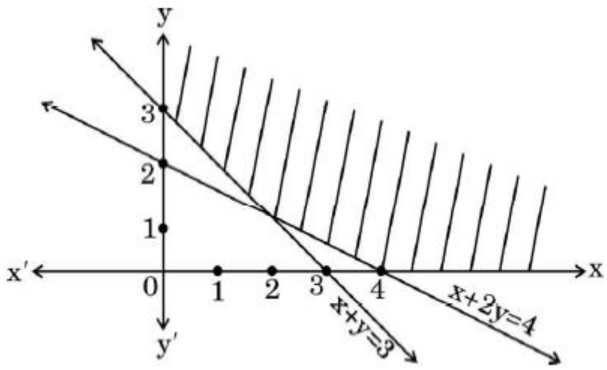
a) z is maximum at (2, 72), minimum at (15, 20)
b) z is maximum at (15, 20), minimum at (40, 15)
c) z is maximum at (40, 15), minimum at (15, 20)
d) z is maximum at (40, 15), minimum at (2, 72)

7 $\int \frac{(x-1)}{(x-2)(x-3)}$ dx equals

a) 2log | $x-3$ | – log | $x-2$ | + C   c) log | $x-3$ | – 2log | $x-2$ | + C

b) log | $x-3$ | – log | $x-2$ | + C   d) log | $x-2$ | – log | $x-3$ | + C

8 The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints?

a) x + 2y ≥ 4, x + y ≤ 3, x ≥ 0, y ≥ 0   b) x + 2y ≥ 4, x + y ≥ 3, x ≤ 0, y ≤ 0

c) x + 2y ≤ 4, x + y ≤ 3, x ≥ 0, y ≥ 0   d) x + 2y ≥ 4, x + y ≥ 3, x ≥ 0, y ≥ 0

9 Let $\sin^{-1}(2x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$. Then the value of *'x'* is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) 8

10 If A is a square matrix of order 2 and |A| = - 3, then the value of |5A| is

a) -3   b) -15   c) -75   d) None of these

11 If y = a cos mx + b sin mx, then $\frac{d^2y}{dx^2}$ is

a) m^2y b) $-m^2y$ c) my   d) - my

12 The general solution of the differential equation x dy – (1 + x^2) dx = dx is

a) y = 2x + $\frac{x^3}{3}$ + c   b) y = 2 log x + $\frac{x^3}{3}$ + c

c) y = $\frac{x^2}{2}$ + c   d) y = 2 log x + $\frac{x^2}{2}$ + c

13 If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is:

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) None of these

14 In ΔABC, $\overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. If D is the mid-point of BC, then $\overrightarrow{AD}$ is equal to

a) $4\hat{\imath} + 6\hat{k}$ b) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ c) $\hat{\imath} - \hat{\jmath} + \hat{k}$ d) $2\hat{\imath} + 3\hat{k}$

15 Find $\boldsymbol{x}$ if $\begin{vmatrix} 3 & -6 \\ 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & x^2 \\ x & -1 \end{vmatrix}$

a) 4 b) √-6 c) -3 d) None of these

16 The value of λ for which the angle between the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k}$ + p ($2\hat{\imath} + \hat{\jmath} + 2\hat{k}$) and $\vec{r}$ = (1 + q)$\hat{\imath}$ + (1 + qλ)$\hat{\jmath}$ + (l + q)$\hat{k}$ is $\frac{\pi}{2}$ is

a) - 4 b) - 2 c) 2 d) 4

17 P(A ∩ B) = $\frac{1}{8}$ and P($\bar{A}$) = $\frac{3}{4}$ then P$\left(\frac{B}{A}\right)$ is equal to

a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

18 If $\vec{a} + \vec{b} = i$ and $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, then $|\vec{b}|$ equals

a) √5 b) √6 c) √9 d) √17

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

19 **Assertion (A):** If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A = I$
Reason (R): AI = IA = A

a) b) c) d)

20 **Assertion (A):** Equation of a line passing through the points (1, 2, 3) and (3, -1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$
Reason (R): Equation of a line passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

a) b) c) d)

SECTION – B

(Each Question Carries 2 Marks)

21 Find the maximum and minimum values of the function given by f(x) = 5 + sin 2x.

22 If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, then find the relation between α and β.

- OR -

The equations of a line are 5x - 3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes

23 Find the value of $sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

- OR -

Find the domain of y = $\sin^{-1}(x^2 - 4)$

24 Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

25 Find the value of '*a*' and '*b*' such that the function defined is a continuous function

$$f(x) = \begin{cases} 5, & if\ x \le 2 \\ ax + b, & if\ 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$$

SECTION – C

(Each Question Carries 3 Marks)

26 Integrate the function $\int \frac{x^2+1}{x^2-5x+6}\,dx$

- OR -

Integrate the function $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$

27 A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

- OR -

There are two coins. One of them is a biased coin such that P(head) : P(tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

28 Evaluate $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx$

29 Solve the following linear programming problem graphically:
Minimise: z = - 3x + 4y
subject to the constraints $x + 2y \le 8$, $3x + 2y \le 12$, $x, y \ge 0$.

30 Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx$

- OR -

Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{2x} . \left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$

31 If y = 3 cos (log x) + 4 sin (log x), show that $x^2 y_2 + xy_1 + y = 0$

SECTION – D

(Each Question Carries 5 Marks)

32 Define the relation R in the set $N \times N$ as follows:
For (a, b), (c, d) ∈ $N \times N$, (a, b) R (c, d) iff ad = bc.
Prove that R is an equivalence relation in $N \times N$.

- OR -

Show that the function f: R → {x ∈ R: - 1< x < 1} defined by f(x) = $\frac{x}{1+|x|}$,
x ∈ R is a one-one onto function

33 The equilibrium conditions for three competitive markets are described as given below, where m_1, m_2 and m_3 are the equilibrium price for each market respectively.
$m_1 + 2m_2 + 3m_3 = 85$
$3m_1 + 2m_2 + 2m_3 = 105$
$2m_1 + 3m_2 + 2m_3 = 110$
Using matrix method, find the values of respective equilibrium prices

34 Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.

- OR -

Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

35 Find the value of *'b'* so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines

- OR -

Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(-1, -2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

SECTION – E

(CASE STUDY - Each Question Carries 4 Marks)

36 Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = -0.1x^2 + mx + 98.6$, $0 \le x \le 12$,
m being a constant, where f(x) is the temperature in F^0 at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer (1m)

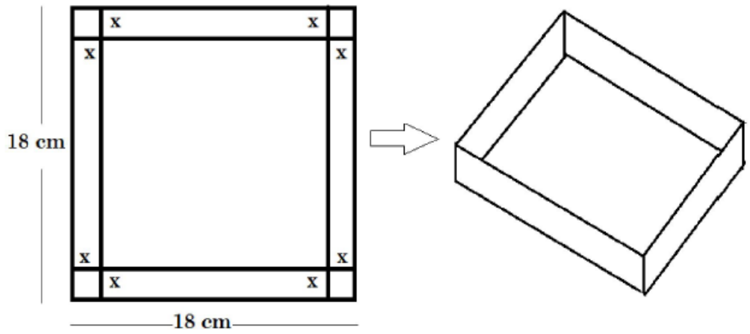
(ii) If 6 is the critical point of the function, then find the value of the constant *m* (1m)

(iii) Find the intervals in which the function is strictly increasing / strictly decreasing. (2m)

\- OR -

Find the points of local maximum / local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum / absolute minimum in the interval [0, 12]. (2m)

37 For an EMC project, a student of Class XII makes an open cardboard box for a jewelry shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps. Assume that 'x' be the side of squares cut off from each corner. Based on the given information, answer the following questions.



(i) For the open box, find the length, breadth and height in terms of x. (1m)

(ii) Write an expression for the volume of the open box. (1m)

(iii) For what value of 'x', the open box will have maximum volume? (2m)

\- OR -

Find the maximum value of volume of the open box. (2m)

38 A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike, the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let, E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions:

(i) What is the probability that many workers were not present given that the construction work was completed on time? (2m)

(ii) What is the probability that all workers were present given that the construction job was completed on time? (2m)