



COMMON PRE-BOARD EXAMINATION 2023-24
Subject: MATHEMATICS (041)
Class XII
MARKING SCHEME



1	b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$	
2	c) $k = 1$	
3	c) $\frac{-8}{3}$	
4	c) $\frac{1}{2} \log 2$	
5	b) 3	
6	c) z is maximum at (40, 15), minimum at (15, 20)	
7	a) $2\log x-3 - \log x-2 + C$	
8	d) $x+2y \geq 4, x+y \geq 3, x \geq 0, y \geq 0$	
9	b) $\frac{1}{4}$	
10	c) -75	
11	b) $-m^2 y$	
12	d) $y = 2 \log x + \frac{x^2}{2} + c$	
13	a) $\frac{\pi}{2}$	
14	d) $2\hat{i} + 3\hat{k}$	
15	c) -3	
16	a) -4	
17	d) $\frac{1}{2}$	
18	c) $\sqrt[3]{9}$	
19	(A) Both A and R are true and R is the correct explanation of A	
20	(D) A is false and R is True	
21	We know that $-1 \leq \sin 2x \leq 1$ $4 \leq \sin 2x + 5 \leq 1 + 5$ $4 \leq \sin 2x + 5 \leq 6$ So, maximum value is 6 and minimum value is 4	
22	$\cos \frac{\pi}{4} = \frac{ \alpha \cdot 1 + 0 + \beta }{\sqrt{\alpha^2 + \beta^2 + 25\sqrt{2}}}$ $\Rightarrow \frac{1}{\sqrt{2}} = \frac{ \alpha + \beta }{\sqrt{\alpha^2 + \beta^2 + 25\sqrt{2}}}$ <p>Squaring both sides, we get</p> $\alpha^2 + \beta^2 + 2\alpha\beta = \alpha^2 + \beta^2 + 25$ $\Rightarrow \alpha\beta = \frac{25}{2}$ <p style="text-align: center;">OR</p>	The equation of the given line is $\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$ Its direction ratios are. $\left(\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}\right)$ or $(6, 2, -3)$ Direction cosines are $(\pm \frac{6}{7}, \pm \frac{2}{7}, \mp \frac{3}{7})$

23	$\begin{aligned} \sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right) &= \sin^{-1}\cos\left(6\pi + \frac{3\pi}{5}\right) = \sin^{-1}\cos\left(\frac{3\pi}{5}\right) = \sin^{-1}\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) \\ &= \frac{\pi}{2} - \frac{3\pi}{5} = -\frac{\pi}{10}. \end{aligned}$ <p>OR</p> $\begin{aligned} -1 \leq (x^2 - 4) \leq 1 \Rightarrow 3 \leq x^2 \leq 5 \Rightarrow \sqrt{3} \leq x \leq \sqrt{5} \\ \Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]. \text{ So, required domain is } [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]. \end{aligned}$														
24	<p>Vector equation of the line passing through $(2, 1, 3)$ is</p> $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$ <p>Line \vec{r} is perpendicular to the given lines then</p> $a + 2b + 3c = 0 ; -3a + 2b + 5c = 0$ $\Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = k' \text{ (say)}$ $\Rightarrow a = 2k, b = -7k \text{ and } c = 4k.$ <p>Thus, the required vector equation is</p> $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$														
25	$x = 2.$ <p>Therefore, LHL = RHL = $f(2)$</p> $\Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$ $\Rightarrow \lim_{x \rightarrow 2^-} 5 = \lim_{x \rightarrow 2^+} ax + b = 5$ $\Rightarrow 2a + b = 5 \dots(i)$														
	$x = 10.$ <p>Therefore, LHL = RHL = $f(10)$</p> $\Rightarrow \lim_{x \rightarrow 10^-} f(x) = \lim_{x \rightarrow 10^+} f(x) = f(10)$ $\Rightarrow \lim_{x \rightarrow 10^-} ax + b = \lim_{x \rightarrow 10^+} 21 = 21$ $\Rightarrow 10a + b = 21 \dots(ii)$ <p>Solving the equation (i) and (ii), we get $a = 2, b = 1$</p>														
26	$\frac{x^2 + 1}{x^2 - 5x + 6} = 1 + \frac{5x - 5}{x^2 - 5x + 6}$ $= 1 + \frac{5x - 5}{(x-2)(x-3)}$ $\frac{5x - 5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$ $5x - 5 = A(x-3) + B(x-2)$														
	<p>Equating the coefficients of x $A + B = 5$</p> <p>Equating the constant terms $3A + 2B = 5$</p> <p>Solving $A = -5$ and $B = 10$</p> $\begin{aligned} \int \frac{x^2 + 1}{x^2 - 5x + 6} dx &= \int dx - 5 \int \frac{1}{x-2} dx + 10 \int \frac{dx}{x-3} \\ &= x - 5 \log x-2 + 10 \log x-3 + C. \end{aligned}$														
	<p>- OR -</p> <p>Let $e^x = t$, so that $e^x dx = dt$. Then,</p> $I = \int \frac{dt}{\sqrt{5 - 4t - t^2}} = \int \frac{dt}{\sqrt{3^2 - (t+2)^2}} = \sin^{-1}\left(\frac{t+2}{3}\right) + C = \sin^{-1}\left(\frac{e^x + 2}{3}\right) + C$														
27	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>X</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr> </thead> <tbody> <tr> <th>P(X)</th><td>$\frac{6}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{8}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{2}{36}$</td></tr> </tbody> </table>	X	0	1	2	3	4	5	P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$
X	0	1	2	3	4	5									
P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$									

- OR -

$$E_1 = \text{Biased coin is selected} \Rightarrow P(E_1) = \frac{1}{2} \quad E_2 = \text{Fair coin is selected} \Rightarrow P(E_2) = \frac{1}{2}$$

$$A = \text{Head appeared on tossing a selected coin.} \quad P\left(\frac{A}{E_1}\right) = \frac{1}{4}, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{2}$$

$$\text{By Bayes' Theorem } P\left(\frac{E_1}{A}\right) = \frac{P(E_1) P\left(\frac{A}{E_1}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2} \cdot \frac{1}{4}}{\frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2}} = \frac{1}{3}$$

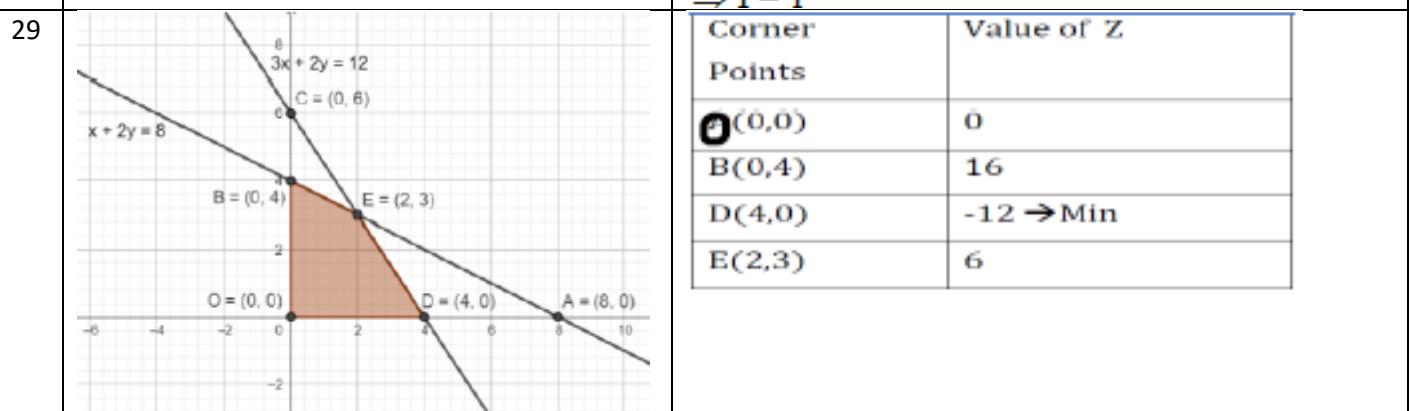
28 Let $I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$

$$I = \int_1^3 \frac{\sqrt{4-(4-x)}}{\sqrt{4-x} + \sqrt{4-(4-x)}} dx$$

$$2I = \int_1^3 \frac{\sqrt{4-x} + \sqrt{x}}{\sqrt{4-x} + \sqrt{x}} dx = \int_1^3 1 dx$$

$$2I = x \Big|_1^3 = 2$$

$$\Rightarrow I = 1$$



30 Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{-(-\sin 2x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1+1-2 \sin x \cos x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{1-(\sin^2 x + \cos^2 x - 2 \sin x \cos x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x) dx}{\sqrt{1-(\sin x - \cos x)^2}}$$

Let $(\sin x - \cos x) = t \Rightarrow (\sin x + \cos x)dx = dt$
When $x = \frac{\pi}{6}$, $t = \left(\frac{1-\sqrt{3}}{2}\right)$,
when $x = \frac{\pi}{3}$, $t = \left(\frac{\sqrt{3}-1}{2}\right)$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$I = \int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$I = 2 \int_0^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}} = [2 \sin^{-1} t]_0^{\frac{\sqrt{3}-1}{2}} = 2 \sin^{-1} \left(\frac{\sqrt{3}-1}{2}\right)$$

- OR -
Put $2x = t$ so that $2 dx = dt$

When $x = \frac{\pi}{2}$, $t = \pi$; $x = \frac{\pi}{4}$, $t = \frac{\pi}{2}$

$$I = \int_{\pi/2}^{\pi} e^t \left(\frac{1 - \sin t}{1 - \cos t} \right) \frac{dt}{2}$$

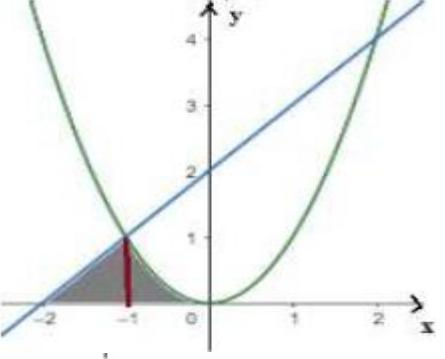
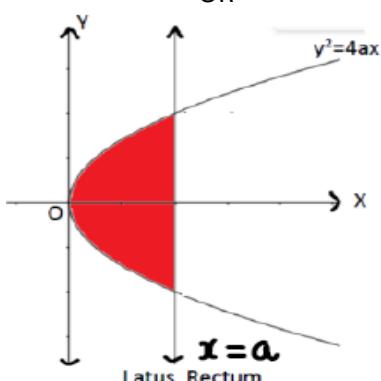
$$= \int_{\pi/2}^{\pi} e^t \left(\frac{1 - 2 \sin t/2 \cos t/2}{2 \sin^2 t/2} \right) \frac{dt}{2}$$

$$= \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} e^t \left(\frac{1}{2} \cosec^2 \frac{t}{2} - \cot \frac{t}{2} \right) dt$$

$$= -\frac{1}{2} \left[e^t \cot \frac{t}{2} \right]_{\pi/2}^{\pi}$$

$$= \frac{1}{2} e^{\pi/2}.$$

31	<p>Given that $y = 3 \cos(\log x) + 4 \sin(\log x)$</p> $\frac{dy}{dx} = \frac{d}{dx}(3 \cos(\log x) + 4 \sin(\log x)) = -3 \sin(\log x) \cdot \frac{1}{x} + 4 \cos(\log x) \cdot \frac{1}{x}$ $\Rightarrow x \frac{dy}{dx} = -3 \sin(\log x) + 4 \cos(\log x)$ $x \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx} x = \frac{d}{dx}[-3 \sin(\log x) + 4 \cos(\log x)]$ $= -3 \cos(\log x) \cdot \frac{1}{x} - 4 \sin(\log x) \cdot \frac{1}{x} = -\frac{1}{x}[3 \cos(\log x) + 4 \sin(\log x)] = -\frac{1}{x} \cdot y$ $\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x} y$ $\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$ $\Rightarrow x^2 y_2 + x y_1 + y = 0$								
32	<p>Let $(a, b) \in N \times N$. Then we have</p> <p>$ab = ba$ (by commutative property of multiplication of natural numbers)</p> <p>$\Rightarrow (a, b)R(a, b)$ Hence, R is reflexive.</p> <p>Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$. Then $ad = bc$</p> <p>$\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers)</p> <p>$\Rightarrow (c, d)R(a, b)$ Hence, R is symmetric</p> <p>Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$.</p> <p>Then $ad = bc, cf = de$</p> <p>$\Rightarrow adcf = bcde$</p> <p>$\Rightarrow af = be$</p> <p>$\Rightarrow (a, b)R(e, f)$</p> <p>Hence, R is transitive.</p> <p>Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.</p> <p>- OR -</p>								
	<table border="0" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 25%; vertical-align: top;"> $x = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases} \rightarrow f(x) = \begin{cases} \frac{x}{1+x}, & x \geq 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$ <p>For $x \geq 0$</p> $f(x_1) = \frac{x_1}{1+x_1}$ $f(x_2) = \frac{x_2}{1+x_2}$ $f(x_1) = f(x_2)$ $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$ $x_1(1+x_2) = x_2(1+x_1)$ $x_1 + x_1x_2 = x_2 + x_2x_1$ $x_1 = x_2$ </td> <td style="width: 25%; vertical-align: top;"> <p>For $x < 0$</p> $f(x_1) = \frac{x_1}{1-x_1}$ $f(x_2) = \frac{x_2}{1-x_2}$ <p>Putting $f(x_1) = f(x_2)$</p> $\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$ $x_1(1-x_2) = x_2(1-x_1)$ $x_1 - x_1x_2 = x_2 - x_2x_1$ $x_1 = x_2$ </td> <td style="width: 25%; vertical-align: top;"> <p>For $x \geq 0$</p> $f(x) = \frac{x}{1+x}$ <p>Let $f(x) = y$,</p> $y = \frac{x}{1+x}$ $x = \frac{y}{1-y}$, for $x \geq 0$ </td> <td style="width: 25%; vertical-align: top;"> <p>For $x < 0$</p> $f(x) = \frac{x}{1-x}$ <p>Let $f(x) = y$</p> $y = \frac{x}{1-x}$ $x = \frac{y}{1+y}$, for $x < 0$ </td> </tr> <tr> <td colspan="2"></td> <td colspan="2"> <p>Here, $y \in \{x \in \mathbb{R}: -1 < x < 1\}$</p> <p>So, x is defined for all values of y.</p> <p>$\therefore f$ is onto</p> <p>Hence, f is one-one and onto.</p> </td> </tr> </table> <p>Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2 \quad \therefore f$ is one-one</p>	$ x = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases} \rightarrow f(x) = \begin{cases} \frac{x}{1+x}, & x \geq 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$ <p>For $x \geq 0$</p> $f(x_1) = \frac{x_1}{1+x_1}$ $f(x_2) = \frac{x_2}{1+x_2}$ $f(x_1) = f(x_2)$ $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$ $x_1(1+x_2) = x_2(1+x_1)$ $x_1 + x_1x_2 = x_2 + x_2x_1$ $x_1 = x_2$	<p>For $x < 0$</p> $f(x_1) = \frac{x_1}{1-x_1}$ $f(x_2) = \frac{x_2}{1-x_2}$ <p>Putting $f(x_1) = f(x_2)$</p> $\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$ $x_1(1-x_2) = x_2(1-x_1)$ $x_1 - x_1x_2 = x_2 - x_2x_1$ $x_1 = x_2$	<p>For $x \geq 0$</p> $f(x) = \frac{x}{1+x}$ <p>Let $f(x) = y$,</p> $y = \frac{x}{1+x}$ $x = \frac{y}{1-y}$, for $x \geq 0$	<p>For $x < 0$</p> $f(x) = \frac{x}{1-x}$ <p>Let $f(x) = y$</p> $y = \frac{x}{1-x}$ $x = \frac{y}{1+y}$, for $x < 0$			<p>Here, $y \in \{x \in \mathbb{R}: -1 < x < 1\}$</p> <p>So, x is defined for all values of y.</p> <p>$\therefore f$ is onto</p> <p>Hence, f is one-one and onto.</p>	
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		<p>Here, $y \in \{x \in \mathbb{R}: -1 < x < 1\}$</p> <p>So, x is defined for all values of y.</p> <p>$\therefore f$ is onto</p> <p>Hence, f is one-one and onto.</p>							

33	$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix}$ $\Rightarrow A = 9 \Rightarrow A^{-1} \text{ exists}$ $\text{And } A^{-1} = \frac{1}{9} \begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}$	$AX = B \Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{9} \begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix} \begin{bmatrix} 85 \\ 105 \\ 110 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$ $\Rightarrow p_1 = 15, p_2 = 20, p_3 = 10$
34	<p>Let Correct Graph :</p>  <p>x coordinates of point of intersection are -1, 2</p> <p>Required area = $\int_{-2}^{-1} (x+2) dx + \int_{-1}^0 x^2 dx$</p> $= \frac{(x+2)^2}{2} \Big _{-2}^{-1} + \frac{x^3}{3} \Big _{-1}^0$ $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$	<p>- OR -</p>  <p>Required area = $2 \int_0^a 2\sqrt{ax} dx$</p> $= 4\sqrt{a} \int_0^a \sqrt{x} dx$ $= 4\sqrt{a} \left \frac{2x^{3/2}}{3} \right _0^a$ $= \frac{8}{3} a \sqrt{a} \sqrt{a}$ $= \frac{8}{3} a^2$
35	<p>As lines are intersecting, $(\vec{a}_2 - \vec{a}_1) \cdot ((\vec{b}_1 \times \vec{b}_2)) = 0$</p> $\Rightarrow \begin{vmatrix} 3 & 1-b & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = 0 \Rightarrow b = 2$ <p>Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$, $\lambda \in \mathbb{R}$</p> <p>For the point of intersection, this point must lie on the line $\frac{x-4}{5} = \frac{y-1}{2} = z$</p> $\Rightarrow \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = 4\lambda + 3 \Rightarrow \lambda = -1$ point of intersection is $(-1, -1, -1)$	
	<p>- OR -</p> <p>Equation of the line AB : $\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$</p> <p>Equation of the line BC : $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$</p> <p>Equation of the line CD : $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$</p> <p>Equation of the line DA : $\frac{x-4}{3} = \frac{y-7}{5} = \frac{z-8}{3}$</p>	

Let P be foot of perpendicular from A to CD.

\therefore Coordinates of P are $(\lambda - 1, 2\lambda - 2, 2\lambda + 1)$ for some λ

d.r.'s of AP are $(\lambda - 5, 2\lambda - 9, 2\lambda - 7)$ since $AP \perp CD$

$$\Rightarrow 1(\lambda - 5) + 2(2\lambda - 9) + 2(2\lambda - 7) = 0 \quad \Rightarrow 9\lambda = 37 \quad \Rightarrow \lambda = \frac{37}{9}$$

\therefore Coordinates of P are $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$

36 (i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in $(0, 12)$

$$(ii) f'(x) = -0.2x + m$$

Since, 6 is the critical point,

$$f'(6) = 0 \Rightarrow m = 1.2$$

$$(iii) f(x) = -0.1x^2 + 1.2x + 98.6$$

$$f'(x) = -0.2x + 1.2 = -0.2(x - 6)$$

In the Interval	$f'(x)$	Conclusion
$(0, 6)$	+ve	f is strictly increasing in $[0, 6]$
$(6, 12)$	-ve	f is strictly decreasing in $[6, 12]$

OR

$$(iii) f(x) = -0.1x^2 + 1.2x + 98.6,$$

$$f'(x) = -0.2x + 1.2, f'(6) = 0,$$

$$f''(x) = -0.2$$

$$f''(6) = -0.2 < 0$$

Hence, by second derivative test 6 is a point of local maximum.

37 (i) For the open box the length, breadth and height is given by $(18 - 2x)$ cm, $(18 - 2x)$ cm and x cm respectively.

(ii) Therefore, the volume of box is, $V = (18 - 2x)(18 - 2x)(x) = (324x - 72x^2 + 4x^3)$ cm³

$$(iii) \text{ Now } \frac{dV}{dx} = 324 - 144x + 12x^2 \text{ and } \frac{d^2V}{dx^2} = -144 + 24x$$

$$\text{For } \frac{dV}{dx} = 0, 12(x^2 - 12x + 27) = 0$$

$$\Rightarrow (x-9)(x-3) = 0$$

$$\text{Either } (x-9) = 0 \text{ or, } (x-3) = 0$$

$$\therefore x \neq 9 \quad \therefore x = 3 \text{ cm}$$

$$\therefore \left(\frac{d^2V}{dx^2} \right)_{at \ x=3} = -144 + 24(3) = -72 < 0$$

So, V is maximum at $x = 3$ cm.

OR

(iii) Refer the solution of (iii) as shown above.

Clearly, the maximum volume of open box will be $V = (18 - 2x)(18 - 2x)(x) = (18 - 6)^2(3)$

$$\Rightarrow V = 432 \text{ cm}^3.$$

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$$P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$$

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) = 0.65 \times 0.35 + 0.35 \times 0.8 = 0.35 \times 1.45 \\ = 0.51$$

$$(i) \quad P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)} = \frac{0.65 \times 0.35}{0.51} = 0.45$$

$$(ii) \quad P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)} = \frac{0.35 \times 0.8}{0.51} = 0.55$$