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CODE NUMBER	041/1/1
SET NUMBER	1



**INDIAN SCHOOL MUSCAT  
FIRST PRE-BOARD EXAMINATION 2023  
MATHEMATICS (041)**



CLASS : XII  
DATE: 04-12-2023

TIME ALLOTTED : 3 HRS.  
MAXIMUM MARKS: 80

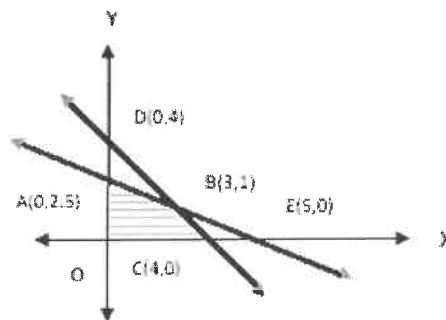
**GENERAL INSTRUCTIONS:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts
6. Section E has 4 Long Answer (LA)-type questions of 5 marks each.

**Section –A (Multiple Choice Questions)  
Each question carries 1 mark**

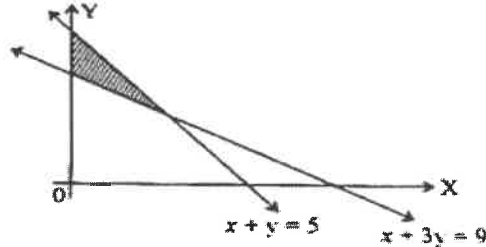
1. Besides non negativity constraint the figure given below is subject to which of the following constraints.

- (a)  $x + 2y \leq 5; x + y \leq 4$
- (b)  $x + 2y \geq 5; x + y \leq 4$
- (c)  $x + 2y \geq 5; x + y \geq 4$
- (d)  $x + 2y \leq 5; x + y \geq 4$



2. If random variable X represents the number of heads when a coin is tossed twice then mathematical expectation of X is  
 (a) 0                      (b)  $\frac{1}{4}$                       (c)  $\frac{1}{2}$                       (d) 1
3. If two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = 2$  and  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$ , then  $|\vec{a} - \vec{b}|$  is  
 (a) -1                      (b)  $\sqrt{8}$                       (c)  $\sqrt{13}$                       (d)  $\sqrt{5}$
4. The set of points of discontinuity of the function  $f(x) = x - [x]$ , is  
 (a) Q                      (b) R                      (c) N                      (d) Z

*Rajeev*

5. If  $f(x) = x \tan^{-1}x$ , then  $f'(1)$  is equal to  
 (a)  $\frac{1}{2} + \frac{\pi}{4}$  (b)  $-\frac{1}{2} + \frac{\pi}{4}$  (c)  $-\frac{1}{2} - \frac{\pi}{4}$  (d)  $\frac{1}{2} - \frac{\pi}{4}$
6. Find the value of:  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$   
 (a) 0 (b) 1 (c) 2 (d) 3
7. For which of the values of  $x$ , the rate of increase of the function  $y = 3x^2 - 2x + 7$  is 4 times the rate of increase of  $x$ ?  
 (a) 0.33 (b) 1 (c) 0.67 (d) -1
8. The value of  $\sin^{-1}(\cos \frac{\pi}{9})$  is  
 (a)  $\frac{\pi}{9}$  (b)  $\frac{5\pi}{9}$  (c)  $\frac{-5\pi}{9}$  (d)  $\frac{7\pi}{18}$
9. For what value of  $x$  is  $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ ?  
 (a) 4 (b) -3 (c) 2 (d) -1
10. In the given figure, what is the LPP shaded region known as?  
 a) Feasible region  
 b) Feasible solution  
 c) Optimal region  
 d) Objective region
- 
11. The angle between the vectors  $\vec{a}$  and  $\vec{b}$  if  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$  is  
 (a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{6}$
12. Integrating factor of the differential equation  $\frac{dy}{dx} + y \tan x - \sec x = 0$  is  
 (a)  $\cos x$  (b)  $\sec x$  (c)  $e^{\cos x}$  (d)  $e^{\sec x}$
13. The maximum value of  $\sin x \cos x$  is  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c)  $\sqrt{2}$  (d)  $2\sqrt{2}$
14. If  $\alpha, \beta, \gamma$  are the direction angles of a vector and  $\cos \alpha = \frac{14}{15}$ ,  $\cos \beta = \frac{1}{3}$ , then  $\cos \gamma =$   
 (a)  $\pm \frac{2}{15}$  (b)  $\pm \frac{1}{5}$  (c)  $\pm \frac{1}{15}$  (d)  $\pm \frac{4}{15}$
15. The total revenue in rupees received from the sale of  $x$  units of a product is given by  $R(x) = 3x^2 + 36x + 5$ . The Marginal revenue, when  $x = 15$  is  
 (a) 116 (b) 96 (c) 90 (d) 126

16. The degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} + 4 - 3\frac{dy}{dx} = 0$
- (a) 2 (b) 1 (c) 3 (d)  $\frac{2}{3}$
17. The rate of change of area of a circle with respect to its radius  $r$  at  $r = 6$  cm is
- (a)  $10\pi$  (b)  $12\pi$  (c)  $8\pi$  (d)  $11\pi$
18. Find the equation of a line passing through  $(1, 2, -3)$  and parallel to the line
- $$\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$$
- (a)  $\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$  (b)  $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$
- (c)  $\frac{x+1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$  (d)  $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-1}{1}$

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).  
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).  
 (c) (A) is true but (R) is false.  
 (d) (A) is false but (R) is true

19. **Assertion (A):** The maximum profit that a company makes if profit function is given by  $P(x) = 41 + 24x - 8x^2$ ; where 'x' is the number of units and P is the profit in rupees is 59.  
**Reason (R):** The profit is maximum at  $x = a$ , if  $P'(a) = 0$  and  $P''(a) > 0$ .
20. **Assertion (A):** If  $\int_0^1 (3x^2 + 2x + k)dx = 0$ , then the value of k is -1.  
**Reason (R):**  $\int x^n dx = \frac{x^{n+1}}{n+1}$

### Section -B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) Find a vector of magnitude 9, perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ .  
**OR**  
 (b) If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is  $\sqrt{3}$ .
22. (a) Simplify:  $\sin^{-1}[2x\sqrt{1-x^2}]$ ,  $x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$   
**OR**  
 (b) Evaluate :  $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$
23. Find the general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$

24. A spherical ice-ball melts uniformly. When its radius is 10 cm, determine the rate of change of its volume with respect to the radius.
25. If  $y = a e^{2x} + b e^{-x}$ , then find the value of  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ .

**Section – C**

**[This section comprises of short answer type questions (SA) of 3 marks each]**

26. Evaluate :  $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$

27. Evaluate:  $\int \frac{(x+3)e^x}{(x+5)^3} dx$

28. (a) Solve the following Linear Programming Problem graphically:  
 Minimize :  $Z = x + 2y$ ,  
 Subject to the constraints :  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ,  $x, y \geq 0$ .

**OR**

- (b) Solve the following Linear Programming Problem graphically:  
 Maximize :  $Z = -x + 2y$ ,  
 Subject to the constraints :  $x + y \geq 5$ ,  $x + 2y \geq 6$ ,  $x \geq 3$ ,  $y \geq 0$ .

29. Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$

30. (a) In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

**OR**

- (b) In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.  
 (i) Find the probability that the student reads neither Hindi nor English newspaper.  
 (ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.  
 (iii) If she reads English newspaper, find the probability that she reads Hindi newspaper.

31. (a) Solve the differential equation:  $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$

**OR**

- (b) Find the particular solution of the differential equation  $x \left( \frac{dy}{dx} \right) - y + x \operatorname{cosec} \left( \frac{y}{x} \right) = 0$ , given that  $y = 0$  when  $x = 1$ .

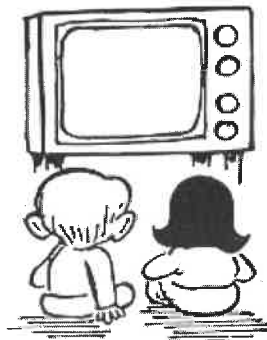
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### Section –D

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

32. Let  $X$  denote the number of hours a person watches television during a randomly selected day. The probability that  $X$  can take the values  $x_i$ , has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$



- (i) Find the value of k.
- (ii) What is the probability that a person watches two hours of television on a selected day?
- (iii) What is the probability that the person watches at least two hours of television on a selected day?

**OR**

What is the probability that the person watches at most two hours of television on a selected day?

33. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the no. of children be  $x$  and the amount distributed by Seema for one child be  $y$  (in ₹). Based on the information given above, answer the following questions:

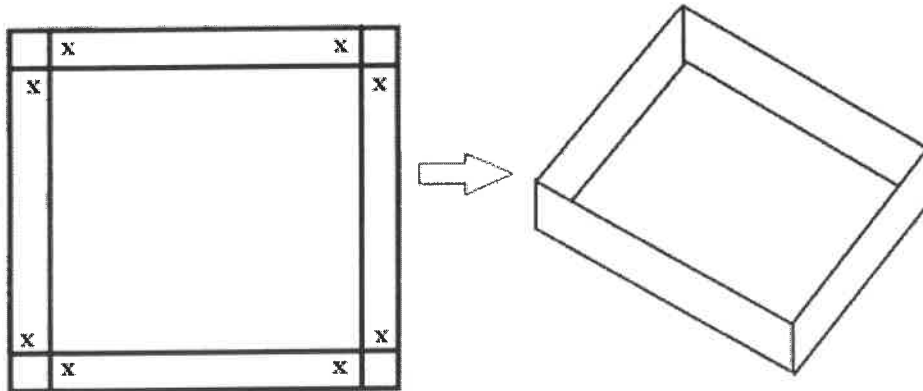


- (i) Write the equations in terms of  $x$  and  $y$ .
- (ii) Represent the matrix equation for the given information.
- (iii) How many children got money from Seema?

**OR**

How much amount Seema has to spend in distributing the money to all the students of orphanage home?

34. Aman has an expensive square shape piece of golden board of size 24 cm which is to be made into a box without top by cutting square of side  $x$  from each corner and folding the flaps to form a box.



- (i) What is the Volume of the open box formed by folding up the flap?
- (ii) In the first derivative test, if  $\frac{dy}{dx}$  changes its sign from positive to negative as  $x$  increases through  $c_1$ , then which value is attained by the function at  $x = c_1$ ?
- (iii) What should be the side of the square piece to be cut from each corner of the board to behold the maximum volume?

**OR**

Find the maximum volume of the box.

### Section –E

[This section comprises of long answer type questions (LA) of 5 marks each]

35. (a) Find the area of  $\Delta ABC$ , the coordinates of whose vertices are  $A(2, 5)$ ,  $B(4, 7)$  and  $C(6, 2)$  using integration.

**OR**

- (b) Using Integration find the area of the region in the first quadrant enclosed by the X-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

36. (a) Find the product  $AB$ , if  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  are two square matrices and hence solve the system of linear equations:  $x - y = 3$ ,  $2x + 3y + 4z = 17$ ,  $y + 2z = 7$  using the product  $AB$ .

**OR**

- (b) If  $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ , then find  $A^{-1}$ . Using  $A^{-1}$ , solve the following system of linear equations  $2x - 3y + 5z = 11$ ,  $3x + 2y - 4z = -5$ ,  $x + y - 2z = -3$

37. Let  $N$  be the set of all natural numbers &  $R$  be the relation on  $N \times N$  defined by  $\{(a, b) R (c, d) \text{ iff } a + d = b + c\}$ . Show that  $R$  is an equivalence relation.
38. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point  $P(5, 4, 2)$  to the line,  $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ . Also find the image of  $P$  in this line.

**\*\*\*\*END OF THE QUESTION PAPER\*\*\*\***



James



ROLL NUMBER				
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CODE NUMBER	041/1/2
SET NUMBER	2



**INDIAN SCHOOL MUSCAT  
FIRST PRE BOARD EXAMINATION 2023  
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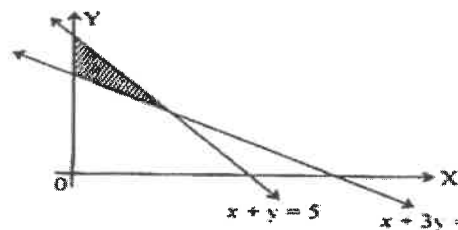
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**Section –A (Multiple Choice Questions)  
Each question carries 1 mark**

1. For which of the values of  $x$ , the rate of increase of the function  $y=3x^2 - 2x+7$  is 4 times the rate of increase of  $x$ ?  
(a) 0-33                      (b) 1                      (c) 0.67                      (d) -1

2. In the given figure , what is the LPP shaded region known as?  
a) Feasible region  
b) Feasible solution  
c) Optimal region  
d) Objective region



3. Find the equation of a line passing through (1, 2, -3) and parallel to the line

$$\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$$

$$(a) \frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$$

$$(b) \frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$$

$$(c) \frac{x+1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$$

$$(d) \frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-1}{1}$$

4. The value of  $\sin^{-1}(\cos \frac{\pi}{9})$  is

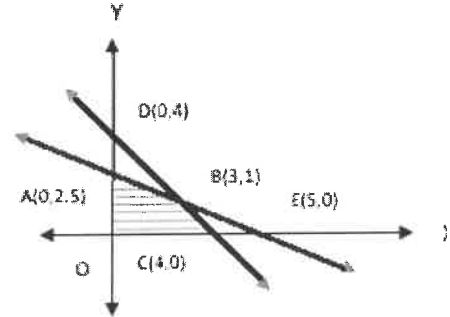
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- (a)  $\frac{\pi}{9}$  (b)  $\frac{5\pi}{9}$  (c)  $\frac{-5\pi}{9}$  (d)  $\frac{7\pi}{18}$

5. The total revenue in rupees received from the sale of  $x$  units of a product is given by  $R(x) = 3x^2 + 36x + 5$ . The Marginal revenue, when  $x = 15$  is  
(a) 116 (b) 96 (c) 90 (d) 126

6. Besides non negativity constraint the figure given below is subject to which of the following constraints.

- (a)  $x + 2y \leq 5; x + y \leq 4$   
(b)  $x + 2y \geq 5; x + y \leq 4$   
(c)  $x + 2y \geq 5; x + y \geq 4$   
(d)  $x + 2y \leq 5; x + y \geq 4$



7. The degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} + 4 - 3\frac{dy}{dx} = 0$

- (a) 2 (b) 1 (c) 3 (d)  $\frac{2}{3}$

8. The set of points of discontinuity of the function  $f(x) = x - [x]$ , is

- (a) Q (b) R (c) N (d) Z

9. Integrating factor of the differential equation  $\frac{dy}{dx} + y \tan x - \sec x = 0$  is

- (a)  $\cos x$  (b)  $\sec x$  (c)  $e^{\cos x}$  (d)  $e^{\sec x}$

10. The maximum value of  $\sin x \cos x$  is

- (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c)  $\sqrt{2}$  (d)  $2\sqrt{2}$

11. If two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = 2$  and  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$ , then  $|\vec{a} - \vec{b}|$  is  
(a) -1 (b)  $\sqrt{8}$  (c)  $\sqrt{13}$  (d)  $\sqrt{5}$

12. Find the value of :  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$

- (a) 0 (b) 1 (c) 2 (d) 3

13. If  $\alpha, \beta, \gamma$  are the direction angles of a vector and  $\cos \alpha = \frac{14}{15}$ ,  $\cos \beta = \frac{1}{3}$ , then  $\cos \gamma =$

- (a)  $\pm \frac{2}{15}$  (b)  $\pm \frac{1}{5}$  (c)  $\pm \frac{1}{15}$  (d)  $\pm \frac{4}{15}$

14. The angle between the vectors  $\vec{a}$  and  $\vec{b}$  if  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$  is

- (a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{6}$

20

15. The rate of change of area of a circle with respect to its radius  $r$  at  $r = 6$  cm is  
 (a)  $10\pi$  (b)  $12\pi$  (c)  $8\pi$  (d)  $11\pi$
16. If random variable  $X$  represents the number of heads when a coin is tossed twice then mathematical expectation of  $X$  is  
 (a) 0 (b)  $\frac{1}{4}$  (c)  $\frac{1}{2}$  (d) 1
17. For what value of  $x$  is  $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ ?  
 (a) 4 (b) -3 (c) 2 (d) -1
18. If  $f(x) = x \tan^{-1}x$ , then  $f'(1)$  is equal to  
 (a)  $\frac{1}{2} + \frac{\pi}{4}$  (b)  $-\frac{1}{2} + \frac{\pi}{4}$  (c)  $-\frac{1}{2} - \frac{\pi}{4}$  (d)  $\frac{1}{2} - \frac{\pi}{4}$

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).  
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).  
 (c) (A) is true but (R) is false.  
 (d) (A) is false but (R) is true
19. **Assertion (A):** If  $\int_0^1 (3x^2 + 2x + k)dx = 0$ , then the value of  $k$  is -1.  
**Reason (R):**  $\int x^n dx = \frac{x^{n+1}}{n+1}$
20. **Assertion (A):** The maximum profit that a company makes if profit function is given by  $P(x) = 41 + 24x - 8x^2$ ; where ' $x$ ' is the number of units and  $P$  is the profit in rupees is 57.  
**Reason (R):** The profit is maximum at  $x = a$ , if  $P'(a) = 0$  and  $P''(a) < 0$ .

### Section -B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) Simplify:  $\sin^{-1}[2x\sqrt{1-x^2}]$ ,  $x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$   
 OR  
 (b) Evaluate :  $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$
22. If  $y = a e^{2x} + b e^{-x}$ , then find the value of  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ .
23. (a) If three non-zero vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$  and  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ , then show that  $\vec{b} = \vec{c}$ .  
 OR  
 (b) Find a vector of magnitude 9, perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ .

24. Find the general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$
25. Find the intervals in which the function  $f(x) = 4x^3 - 6x^2 - 72x + 30$  is strictly decreasing.

### Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate :  $\int_0^4 |x - 1| dx$
27. (a) In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

OR

- (b) In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.
- (i) Find the probability that the student reads neither Hindi nor English newspaper.
- (ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.
- (iii) If she reads English newspaper, find the probability that she reads Hindi newspaper.

28. Evaluate:  $\int \frac{(x+3)e^x}{(x+5)^3} dx$

29. (a) Solve the differential equation:  $y dx + (x - y^2) dy = 0$

OR

- (b) Find the particular solution of the differential equation  
 $x \left( \frac{dy}{dx} \right) - y + x \operatorname{cosec} \left( \frac{y}{x} \right) = 0$ , given that  $y = 0$  when  $x = 1$ .

30. Evaluate :  $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$

31. (a) Solve the following Linear Programming Problem graphically:  
 Minimize :  $Z = x + 2y$ ,  
 Subject to the constraints :  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ,  $x, y \geq 0$ .

OR

- (b) Solve the following Linear Programming Problem graphically:  
 Maximize :  $Z = -x + 2y$ ,  
 Subject to the constraints :  $x + y \geq 5$ ,  $x + 2y \geq 6$ ,  $x \geq 3$ ,  $y \geq 0$ .

### Section –D

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]



32. On his birthday Shyam decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the no. of children be  $x$  and the amount distributed by Shyam for one child be  $y$  (in ₹). Based on the information given above, answer the following questions:

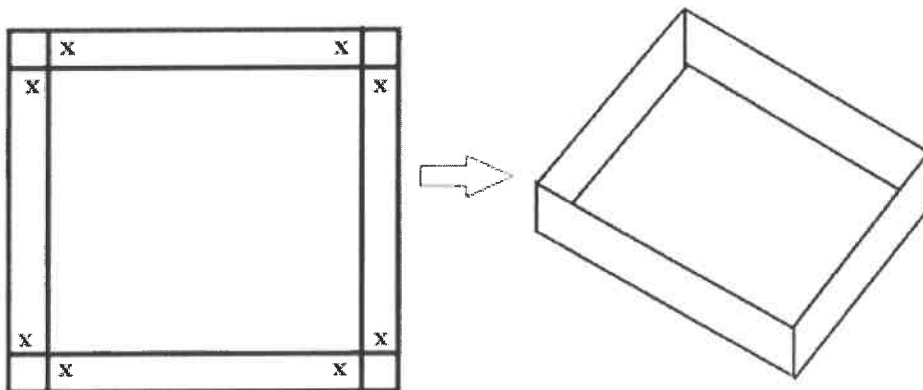


- Write the equations in terms of  $x$  and  $y$ .
- Represent the matrix equation for the given information.
- How many children got money from Shyam?

**OR**

How much amount Shyam has to spend in distributing the money to all the students of orphanage home?

33. Aman has an expensive square shape piece of golden board of size 24 cm which is to be made into a box without top by cutting square of side  $x$  from each corner and folding the flaps to form a box.



- What is the Volume of the open box formed by folding up the flap?
- In the first derivative test, if  $\frac{dy}{dx}$  changes its sign from positive to negative as  $x$  increases through  $c_1$ , then which value is attained by the function at  $x = c_1$ ?
- What should be the side of the square piece to be cut from each corner of the board to behold the maximum volume?

**OR**

Find the maximum volume of the box.

34. Let  $X$  denote the number of hours a person watches television during a randomly selected day. The probability that  $X$  can take the values  $x_i$ , has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$



- (i) Find the value of  $k$ .
- (ii) What is the probability that a person watches two hours of television on a selected day?
- (iii) What is the probability that the person watches at least two hours of television on a selected day?

**OR**

What is the probability that the person watches at most two hours of television on a selected day?

#### Section –E

[This section comprises of long answer type questions (LA) of 5 marks each]

35. (a) Find the product  $AB$ , if  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  are two square matrices and hence solve the system of linear equations:  $x - y = 3$ ,  $2x + 3y + 4z = 17$ ,  $y + 2z = 7$  using the product  $AB$ .

**OR**

- (b) If  $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ , then find  $A^{-1}$ . Using  $A^{-1}$ , solve the following system of linear equations  $2x - 3y + 5z = 11$ ,  $3x + 2y - 4z = -5$ ,  $x + y - 2z = -3$

36. Define the relation  $R$  in the set  $N \times N$  as follows:  
For  $(a, b), (c, d) \in N \times N$ ,  $(a, b) R (c, d)$  iff  $ad = bc$ . Prove that  $R$  is an equivalence relation in  $N \times N$ .
37. (a) Find the area of  $\Delta ABC$ , the coordinates of whose vertices are  $A(2, 5)$ ,  $B(4, 7)$  and  $C(6, 2)$  using integration.
- OR**
- (b) Make a rough sketch of the region  $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$  and find the area of the region using integration.
38. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point  $P(5, 4, 2)$  to the line,  $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ . Also find the image of  $P$  in this line.

\*\*\*\*END OF THE QUESTION PAPER\*\*\*\*

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CODE NUMBER	041/1/3
SET NUMBER	3



**INDIAN SCHOOL MUSCAT  
FIRST PRE-BOARD EXAMINATION 2023  
MATHEMATICS (041)**



CLASS : XII  
DATE: 04-12-2023

TIME ALLOTTED : 3 HRS.  
MAXIMUM MARKS: 80

**GENERAL INSTRUCTIONS:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts
6. Section E has 4 Long Answer (LA)-type questions of 5 marks each.

**Section –A (Multiple Choice Questions)**

**Each question carries 1 mark**

1. Find the value of :  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$   
(a) 0 (b) 1 (c) 2 (d) 3
2. The angle between the vectors  $\vec{a}$  and  $\vec{b}$  if  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$  is  
(a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{6}$
3. The set of points of discontinuity of the function  $f(x) = x - [x]$ , is  
(a) Q (b) R (c) N (d) Z
4. If  $\alpha, \beta, \gamma$  are the direction angles of a vector and  $\cos \alpha = \frac{14}{15}$ ,  $\cos \beta = \frac{1}{3}$ , then  $\cos \gamma =$   
(a)  $\pm \frac{2}{15}$  (b)  $\pm \frac{1}{5}$  (c)  $\pm \frac{1}{15}$  (d)  $\pm \frac{4}{15}$
5. For which of the values of x, the rate of increase of the function  $y=3x^2-2x+7$  is 4 times the rate of increase of x?  
(a) 0-33 (b) 1 (c) 0.67 (d) -1
6. For what value of x is  $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ ?  
(a) 4 (b) -3 (c) 2 (d) -1

*Page 1 of 6*

7. The rate of change of area of a circle with respect to its radius  $r$  at  $r = 6$  cm is  
 (a)  $10\pi$  (b)  $12\pi$  (c)  $8\pi$  (d)  $11\pi$

8. If two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = 2$  and  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$ , then  $|\vec{a} - \vec{b}|$  is  
 (a) -1 (b)  $\sqrt{8}$  (c)  $\sqrt{13}$  (d)  $\sqrt{5}$

9. The degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} + 4 - 3\frac{dy}{dx} = 0$   
 (a) 2 (b) 1 (c) 3 (d)  $\frac{2}{3}$

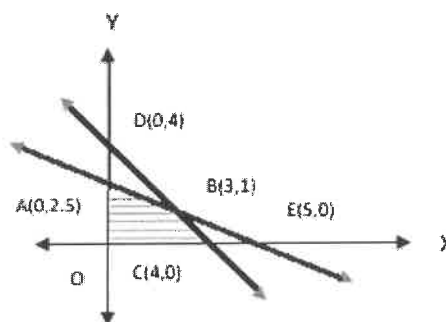
10. The total revenue in rupees received from the sale of  $x$  units of a product is given by  $R(x) = 3x^2 + 36x + 5$ . The Marginal revenue, when  $x = 15$  is  
 (a) 116 (b) 96 (c) 90 (d) 126

11. The maximum value of  $\sin x \cos x$  is  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c)  $\sqrt{2}$  (d)  $2\sqrt{2}$

12. The value of  $\sin^{-1}(\cos \frac{\pi}{9})$  is  
 (a)  $\frac{\pi}{9}$  (b)  $\frac{5\pi}{9}$  (c)  $\frac{-5\pi}{9}$  (d)  $\frac{7\pi}{18}$

13. Besides non negativity constraint the figure given below is subject to which of the following constraints.

- (a)  $x + 2y \leq 5; x + y \leq 4$   
 (b)  $x + 2y \geq 5; x + y \leq 4$   
 (c)  $x + 2y \geq 5; x + y \geq 4$   
 (d)  $x + 2y \leq 5; x + y \geq 4$



14. Integrating factor of the differential equation  $\frac{dy}{dx} + y \tan x - \sec x = 0$  is

- (a)  $\cos x$  (b)  $\sec x$  (c)  $e^{\cos x}$  (d)  $e^{\sec x}$

15. If  $f(x) = x \tan^{-1}x$ , then  $f'(1)$  is equal to

- (a)  $\frac{1}{2} + \frac{\pi}{4}$  (b)  $-\frac{1}{2} + \frac{\pi}{4}$  (c)  $-\frac{1}{2} - \frac{\pi}{4}$  (d)  $\frac{1}{2} - \frac{\pi}{4}$

16. Find the equation of a line passing through  $(1, 2, -3)$  and parallel to the line

$$\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$$



$$(a) \frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$$

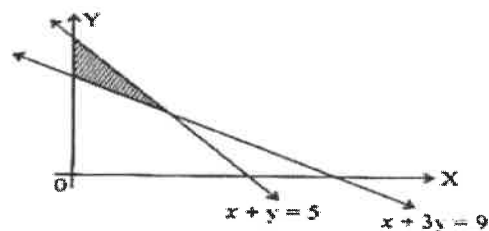
$$(b) \frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$$

$$(c) \frac{x+1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$$

$$(d) \frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-1}{1}$$

17. In the given figure, what is the LPP shaded region known as?

- a) Feasible region  
b) Feasible solution  
c) Optimal region  
d) Objective region



18. If random variable X represents the number of heads when a coin is tossed twice then mathematical expectation of X is

- (a) 0                      (b)  $\frac{1}{4}$                       (c)  $\frac{1}{2}$                       (d) 1

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).  
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).  
(c) (A) is true but (R) is false.  
(d) (A) is false but (R) is true

19. **Assertion (A):** The maximum profit that a company makes if profit function is given by  $P(x) = 41 + 24x - 8x^2$ ; where 'x' is the number of units and P is the profit in rupees is 59.

**Reason (R):** The profit is maximum at  $x = a$ , if  $P'(a) = 0$  and  $P''(a) < 0$ .

20. **Assertion (A):** If  $\int_0^1 (3x^2 + 2x + k) dx = 0$ , then the value of k is -2.

**Reason (R):**  $\int x^n dx = \frac{x^{n+1}}{n+1}$

### Section -B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$
22. Find the intervals in which the function  $f(x) = 4x^3 - 6x^2 - 72x + 30$  is strictly increasing.
23. (a) Simplify:  $\sin^{-1}[2x\sqrt{1-x^2}]$ ,  $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$   
OR  
(b) Evaluate :  $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \cot^{-1}(1)$
24. If  $y = a e^{2x} + b e^{-x}$ , then find the value of  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ .
25. (a) Find a vector of magnitude 9, perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ .

20

OR

- (b) If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is  $\sqrt{3}$ .

**Section – C**

**[This section comprises of short answer type questions (SA) of 3 marks each]**

26. (a) In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.  
(i) Find the probability that the student reads neither Hindi nor English newspaper.  
(ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.  
(iii) If she reads English newspaper, find the probability that she reads Hindi newspaper.

OR

- (b) In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

27. Evaluate:  $\int \frac{(x+3)e^x}{(x+5)^3} dx$

28. Evaluate :  $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$

29. (a) Solve the following Linear Programming Problem graphically:  
Minimize :  $Z = x + 2y$ ,  
Subject to the constraints :  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ,  $x, y \geq 0$ .

OR

- (b) Solve the following Linear Programming Problem graphically:  
Maximize :  $Z = -x + 2y$ ,  
Subject to the constraints :  $x + y \geq 5$ ,  $x + 2y \geq 6$ ,  $x \geq 3$ ,  $y \geq 0$ .

30. (a) Find the particular solution of the differential equation  
 $x \left( \frac{dy}{dx} \right) - y + x \operatorname{cosec} \left( \frac{y}{x} \right) = 0$ , given that  $y = 0$  when  $x = 1$ .

OR

- (b) Solve the differential equation:  $x dy - y dx = \sqrt{x^2 + y^2} dx$

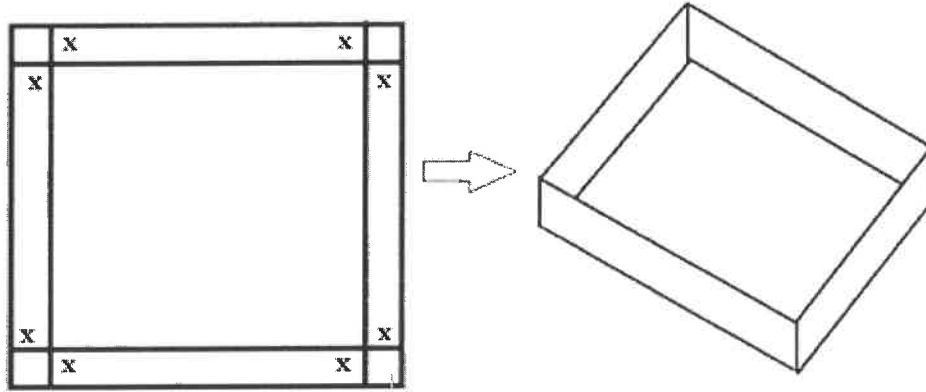
31. Evaluate :  $\int \frac{1}{\sqrt{3-2x-x^2}} dx$

**Section –D**

**[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]**

By

32. Aman has an expensive square shape piece of golden board of size 24 cm which is to be made into a box without top by cutting square of side  $x$  from each corner and folding the flaps to form a box.



- What is the Volume of the open box formed by folding up the flap?
- In the first derivative test, if  $\frac{dy}{dx}$  changes its sign from positive to negative as  $x$  increases through  $c_1$ , then which value is attained by the function at  $x = c_1$ ?
- What should be the side of the square piece to be cut from each corner of the board to behold the maximum volume?

**OR**

Find the maximum volume of the box.

33. Let  $X$  denote the number of hours a person watches television during a randomly selected day. The probability that  $X$  can take the values  $x_i$ , has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$



- Find the value of  $k$ .
- What is the probability that a person watches two hours of television on a selected day?
- What is the probability that the person watches at least two hours of television on a selected day?

**OR**

What is the probability that the person watches at most two hours of television on a selected day?

34. On her birthday Neha decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the no. of children be  $x$  and the amount distributed by Neha for one child be  $y$  (in ₹). Based on the information given above, answer the following questions:



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- Write the equations in terms of  $x$  and  $y$ .
- Represent the matrix equation for the given information.
- How many children got money from Neha?

**OR**

How much amount Neha has to spend in distributing the money to all the students of orphanage home?

### Section –E

**[This section comprises of long answer type questions (LA) of 5 marks each]**

35. Let  $N$  be the set of all natural numbers &  $R$  be the relation on  $N \times N$  defined by  $\{(a, b) R (c, d) \text{ iff } a + d = b + c\}$ . Show that  $R$  is an equivalence relation.
36. (a) Using Integration find the area of the region in the first quadrant enclosed by the  $X$ -axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

**OR**

- (b) Make a rough sketch of the region  $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$  and find the area of the region using integration.

37. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point  $P(5, 4, 2)$  to the line,  $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ . Also find the image of  $P$  in this line.

38. (a) Find the product  $AB$ , if  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  are two square matrices

and hence solve the system of linear equations:  $x - y = 3$ ,  $2x + 3y + 4z = 17$ ,  $y + 2z = 7$

using the product  $AB$ .

**OR**

- (b) If  $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ , then find  $A^{-1}$ . Using  $A^{-1}$ , solve the following system of linear equations  $2x - 3y + 5z = 11$ ,  $3x + 2y - 4z = -5$ ,  $x + y - 2z = -3$

**\*\*\*\*END OF THE QUESTION PAPER\*\*\*\***

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