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| ROLL NUMBER | | | | |
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| SET | A |
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INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
SUBJECT: MATHEMATICS
SUBJECT CODE : 041



CLASS : XII
 DATE: 14-09-2023

TIME ALLOTTED : 3 HRS.
 MAXIMUM MARKS:

GENERAL INSTRUCTIONS:

1. This question paper consists of **five sections A, B, C, D and E**. Each section is compulsory.
2. **Section A** comprises of 18 MCQs of one mark each (from Q01-Q18) and Assertion-Reasoning based questions (from Q 19- Q20)
3. **Section B** comprises of 05 Very Short Answer (VSA)-type questions of 2 marks each (from Q21-Q25) .
4. **Section C** comprises of 06 Short Answer (SA)-type questions of 3 marks each (from Q26-Q31) .
5. **Section D** comprises of 03 Case-study based questions (from Q32-Q34)
6. **Section E** comprises of 04 Long Answer (LA)-type questions of 5 marks each (from Q35-Q38)
7. There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

SECTION-A
(Multiple Choice Questions)
Question numbers 1 to 20 carry 1 mark each

1. Set A has 3 elements and set B has 4 elements. Then, the number of injective mappings that can be defined from A to B is
 (a) 144 (b) 12 (c) 24 (d) 64
2. If $\tan^{-1}x = y$, then
 (a) $-1 < y < 1$ (b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$
3. If A and B are square matrices of the same order 3, such that $|A|=3$ and $AB = 2I$, then the value of $|B|$ is
 (a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) 1
4. The value of θ in $[0, 2\pi]$ such that the matrix $\begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\pi + \theta) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\pi - \theta) & \tan(\pi - \theta) & 0 \end{bmatrix}$ is skew symmetric, is
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

5. The probability of solving the specific problems independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then the probability that exactly one of them solves the problem is
 (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$
6. If two events are independent, then
 (a) They must be mutually exclusive (b) sum of their probabilities must be equal to 1
 (c) (a) and (b) both are correct (d) none of the above is correct
7. The region represented by the inequalities $x, y \geq 0, y \leq 6, x + y \leq 3$ is
 (a) Unbounded in the first quadrant (b) unbounded in the first and second quadrant
 (c) bounded in the first quadrant (d) none of these
8. In an LPP, if the objective function $z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is
 (a) 0 (b) 2 (c) finite (d) infinite
9. A point 'c' in the domain of f is called critical point of 'f', if
 I $f'(c) = 0$.
 II f is not differentiable at c.
 Choose the correct option
 (a) Either I or II are true
 (b) Only I is true
 (c) Only II is true
 (d) Neither I nor II is true
10. If $y = \log \sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by
 (a) ∞ (b) 1 (c) 0 (d) $\frac{1}{2}$
11. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + p, & x < 4 \\ p + q, & x = 4 \\ \frac{x-4}{|x-4|} + q, & x > 4 \end{cases}$ Then $f(x)$ is continuous at $x = 4$ when
 (a) $p = 0, q = 0$ (b) $p = 1, q = 1$ (c) $p = -1, q = 1$ (d) $p = 1, q = -1$
12. The function $f(x) = x^x$ has a stationary point at x equals to
 (a) e (b) $\frac{1}{e}$ (c) 1 (d) \sqrt{e}
13. The function $f(x) = \tan x - x$
 (a) always increases (b) always decreases
 (c) never increases (d) neither increasing nor decreasing
14. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?
 (a) $f(x) + g(x)$ (b) $f(x) - g(x)$ (c) $f(x) \cdot g(x)$ (d) $\frac{g(x)}{f(x)}$
15. The set of points, where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable, is

- (a) R (b) $R - \left\{\frac{1}{2}\right\}$ (c) $(0, \infty)$ (d) none of these
16. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then depth of the wheat is increasing at the rate of
(a) 1 m/h (b) 0.1 m/h (c) 1.1 m/h (d) 0.5 m/h
17. If $y = \log_7(\log x)$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{1}{x \log x \log 7}$ (b) $\frac{-1}{x \log x \log 7}$ (c) $\frac{1}{x \log x}$ (d) none of these
18. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by
- $$a_{ij} = \begin{cases} 2i + 3j, & i < j \\ 5, & i = j \\ 3i - 2j, & i > j \end{cases}$$
- The number of elements in A which are more than 5, is
(a) 3 (b) 4 (c) 5 (d) 6

Following are Assertion-Reasoning based questions (from Q19-Q20):

Read the following statements carefully to mark the correct option out of the options given below.

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.
19. **Assertion (A):** The function $f: \{1,2,3,4\} \rightarrow \{x,y,z,p\}$ defined by $f = \{(1,x), (2,y), (3,z), (4,z)\}$ is a bijective function
Reason (R): The function $f: \{1,2,3\} \rightarrow \{x,y,z,p\}$ such that $f = \{(1,x), (2,y), (3,z)\}$ is one-one.
20. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then
Assertion (A): $f(x)$ has a minimum at $x=1$
Reason (R): When $\frac{d}{dx}(f(x)) < 0 \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0 \forall x \in (a, a+h)$; where 'h' is an infinitely small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

SECTION-B

(Questions 21 to 25 carry 2 marks each)

21. Find the value of $\sin^{-1}[\cos(\frac{3\pi}{5})]$.
OR
Find the domain of the function $\cos^{-1}(\sqrt{x-1})$.
22. For what values of x and y are the following matrices equal?

$$\text{If } A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} \text{ and } B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

23. Differentiate $\sqrt{\tan\sqrt{x}}$ with respect to x .
24. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value.
25. If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.
OR
Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

SECTION-C
(Questions 26 to 31 carry 3 marks each)

26. Let $f: N \rightarrow N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$
Find whether the function f is one-one, onto or bijective.
OR
Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.
27. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence deduce A^{-1} .
OR
Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrices.
28. Find the intervals in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.
29. A and B appear for an interview for two posts. The probability of A's selection is $\frac{1}{3}$ and that of B's selection is $\frac{2}{5}$. Find the probability that
(i) only one of them is selected.
(ii) at least one of them is selected.
30. Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as
 $R = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$. Is R reflexive, symmetric and transitive? Give reasons to the answer.
31. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of number of kings.

OR

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred at random from bag I to bag II and then a ball is drawn from bag II. Find the probability that ball drawn is red.

SECTION-D

(This section comprises of 3 Case study/ passage based questions of 4 marks each with 3 sub-parts (i), (ii), (iii) of marks 1,1,2 respectively.)

32. A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

| Market | Products(in numbers) | | |
|--------|----------------------|--------|-----------|
| | Pencil | Eraser | Sharpener |
| A | 10,000 | 2000 | 18,000 |
| B | 6000 | 20,000 | 8000 |

If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then based on the above information answer the following using matrix algebra :

- (i) Express the total revenue as product of two matrices
- (ii) Express the total cost as product of two matrices.
- (iii) Find the profit in market A.

OR

- (iii) Find the profit in market B.

33. In answering a question on a multiple choice test for class XII , a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, A be the events that the student knows the answer , guesses the answer and answers correctly respectively.



- (i) Find the probability that student answered correctly.
- (ii) Find $\sum_{i=1}^2 P(E_i|A)$
- (iii) Find the probability that the student knows the answer given that he answered it correctly.

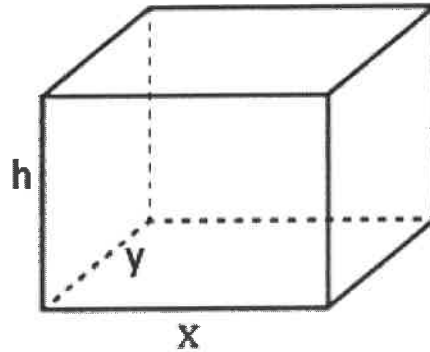
OR

- (iii) Find the probability that the student guesses the answer given that he answered it

34.



metal sheet



To conserve water for emergency needs in a village, the gram panchayat decides to construct an open tank with a square base from a sheet of metal which has been donated by a company. From the account of social welfare account. The area of metal sheet donated is to be least and water to be stored is 4000 m^3

- (i) If x, y and h represent the sides of the base and height of the tank respectively, then find the area A of metal sheet used.
- (ii) Find the area of metal sheet in terms of x only.
- (iii) Area of metal sheet used is least then what will be the length of side of base?

OR

- (iii) Find the least area of metal sheet used.

SECTION-E

(Questions 35 to 38 carry 5 marks each)

35. Show that the relation R defined by $(a, b)R(c, d)$ iff $a + d = b + c$ on $A \times A$, where $A = \{1, 2, 3, \dots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]$; $a, b, c, d \in A$.

OR

Show that the function $f: R \rightarrow R$, defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

36. Differentiate the function $y = x^{\sin x} + \sin x^{\cos x}$ with respect to x .

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

37. Maximize $Z = 5x + 10y$, subject to the constraints,
 $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$
Find also the point(s) at which function attains maximum value

38. Find A^{-1} , where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. Hence solve the following system of equations using Matrix Method, $4x + 2y + 3z = 2, x + y + z = 1, 3x + y - 2z = 5$.

END OF THE QUESTION PAPER

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INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
SUBJECT: MATHEMATICS
SUBJECT CODE : 041



CLASS : XII
 DATE: 14-09-2023

TIME ALLOTTED : 3 HRS.
 MAXIMUM MARKS: 80 marks

GENERAL INSTRUCTIONS:

1. This question paper consists of **five sections A, B, C, D and E**. Each section is compulsory.
2. **Section A** comprises of 18 MCQs of one mark each (from Q01-Q18) and Assertion-Reasoning based questions (from Q 19- Q20)
3. **Section B** comprises of 05 Very Short Answer (VSA)-type questions of 2 marks each (from Q21-Q25) .
4. **Section C** comprises of 06 Short Answer (SA)-type questions of 3 marks each (from Q26-Q31) .
5. **Section D** comprises of 03 Case-study based questions (from Q32-Q34)
6. **Section E** comprises of 04 Long Answer (LA)-type questions of 5 marks each (from Q35-Q38)
7. There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

SECTION-A
(Multiple Choice Questions)
Question numbers 1 to 20 carry 1 mark each

1. If $y = \log \sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by
 (a) ∞ (b) 1 (c) 0 (d) $\frac{1}{2}$
2. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + p, & x < 4 \\ p + q, & x = 4 \\ \frac{x-4}{|x-4|} + q, & x > 4 \end{cases}$ Then $f(x)$ is continuous at $x = 4$ when
 (a) $p = 0, q = 0$ (b) $p = 1, q = 1$ (c) $p = -1, q = 1$ (d) $p = 1, q = -1$
3. The function $f(x) = x^x$ has a stationary point at x equals to
 (a) e (b) $\frac{1}{e}$ (c) 1 (d) \sqrt{e}
4. The function $f(x) = \tan x - x$
 (a) always increases (b) always decreases
 (c) never increases (d) neither increasing nor decreasing
5. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i + 3j, i < j \\ 5, i = j \\ 3i - 2j, i > j \end{cases}$$

The number of elements in A which are more than 5, is

- (a) 3 (b) 4 (c) 5 (d) 6

6. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then depth of the wheat is increasing at the rate of

- (a) 1 m/h (b) 0.1 m/h (c) 1.1 m/h (d) 0.5 m/h

7. If $y = \log_7(\log x)$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{1}{x \log x \log 7}$ (b) $\frac{-1}{x \log x \log 7}$ (c) $\frac{1}{x \log x}$ (d) none of these

8. The set of points, where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable, is

- (a) R (b) $R - \left\{\frac{1}{2}\right\}$ (c) $(0, \infty)$ (d) none of these

9. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

- (a) $f(x) + g(x)$ (b) $f(x) - g(x)$ (c) $f(x) \cdot g(x)$ (d) $\frac{g(x)}{f(x)}$

10. Set A has 3 elements and set B has 4 elements. Then, the number of injective mappings that can be defined from A to B is

- (a) 144 (b) 12 (c) 24 (d) 64

11. If $\tan^{-1}x = y$, then

- (a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$ (c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

12. If A and B are square matrices of the same order 3, such that $|A|=3$ and $AB = 2I$, then the value of $|B|$ is

- (a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) 1

13. The value of θ in $[0, 2\pi]$ such that the matrix $\begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\pi + \theta) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\pi - \theta) & \tan(\pi - \theta) & 0 \end{bmatrix}$ is skew symmetric, is

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

14. The probability of solving the specific problems independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then the probability that exactly one of them solves the problem is

- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

15. If two events are independent, then

- (a) They must be mutually exclusive (b) sum of their probabilities must be equal to 1
(c) (a) and (b) both are correct (d) none of the above is correct

16. The region represented by the inequalities $x, y \geq 0, y \leq 6, x + y \leq 3$ is
 (a) Unbounded in the first quadrant (b) unbounded in the first and second quadrant
 (c) bounded in the first quadrant (d) none of these
17. In an LPP, if the objective function $z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is
 (a) 0 (b) 2 (c) finite (d) infinite
18. A point 'c' in the domain of f is called critical point of 'f', if
 I $f'(c) = 0$.
 II f is not differentiable at c.
 Choose the correct option
 (a) Either I or II are true
 (b) Only I is true
 (c) Only II is true
 (d) Neither I nor II is true

Following are Assertion-Reasoning based questions (from Q19-Q20):

Read the following statements carefully to mark the correct option out of the options given below.

- (a) Both **A** and **R** are true and **R** is the correct explanation of **A**.
 (b) Both **A** and **R** are true but **R** is not the correct explanation of **A**.
 (c) **A** is true but **R** is false.
 (d) **A** is false but **R** is true.

19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then
Assertion (A): $f(x)$ has a minimum at $x=1$
Reason (R): When $\frac{d}{dx}(f(x)) < 0 \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0 \forall x \in (a, a+h)$; where 'h' is an infinitely small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.
20. **Assertion (A):** The function $f: \{1,2,3,4\} \rightarrow \{x,y,z,p\}$ defined by $f = \{(1,x), (2,y), (3,z), (4,z)\}$ is a bijective function
Reason (R): The function $f: \{1,2,3\} \rightarrow \{x,y,z,p\}$ such that $f = \{(1,x), (2,y), (3,z)\}$ is one-one.

SECTION-B

(Questions 21 to 25 carry 2 marks each)

21. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value
22. If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.
 OR
 Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.
23. Find the value of $\sin^{-1}[\cos(\frac{3\pi}{5})]$.
 OR

Find the domain of the function $\sin^{-1}(\sqrt{x-1})$.

24. For what values of x and y are the following matrices equal?

$$\text{If } A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} \text{ and } B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

25. Differentiate $\sqrt{\cot\sqrt{x}}$ with respect to x .

SECTION-C

(Questions 26 to 31 carry 3 marks each)

26. Let $A = \{0,1,2,3\}$ and define a relation R on A as $R = \{(0,0)(0,1)(0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$. Is R reflexive, symmetric and transitive? Give reasons to the answer.

27. A and B appear for an interview for two posts. The probability of A 's selection is $\frac{1}{3}$ and that of B 's selection is $\frac{2}{5}$. Find the probability that
- only one of them is selected.
 - at least one of them is selected.

28. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence deduce A^{-1} .

OR

Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrices.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of number of queens.

OR

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred at random from bag I to bag II and then a ball is drawn from bag II. Find the probability that ball drawn is red.

30. Let $f: N \rightarrow N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$

Find whether the function f is onto, one-one or bijective.

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

31. Find the intervals in which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is strictly increasing or strictly decreasing.

SECTION-D

(This section comprises of 3 Case study/ passage based questions of 4 marks each with 3 sub-parts (i), (ii), (iii) of marks 1,1,2 respectively.)

32. In answering a question on a multiple choice test for class XII , a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, A be the events that the student knows the answer , guesses the answer and answers correctly respectively.



- (i) Find the probability that student answered correctly.
- (ii) Find $\sum_{i=1}^2 P(E_i|A)$
- (iii) Find the probability that the student knows the answer given that he answered it correctly.

OR

- (iii) Find the probability that the student guesses the answer given that he answered it.

33. A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

| Market | Products(in numbers) | | |
|--------|----------------------|--------|-----------|
| | Pencil | Eraser | Sharpener |
| A | 10,000 | 2000 | 18,000 |
| B | 6000 | 20,000 | 8000 |

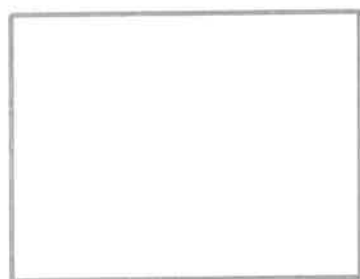
If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then based on the above information answer the following using matrix algebra :

- (i) Express the total revenue as a product of two matrices.
- (ii) Express the total cost as a product of two matrices..
- (iii) Find the profit in market A.

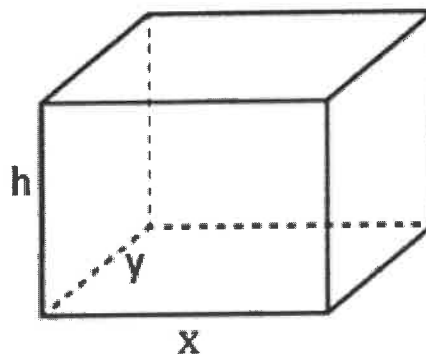
OR

- (iii) Find the profit in market B.

34.



metal sheet



To conserve water for emergency needs in a village, the gram panchayat decides to construct an open tank with a square base from a sheet of metal which has been donated by a company. From the account of social welfare account. The area of metal sheet donated is to be least and water to be stored is 4000 m^3

- (i) If x , y and h represent the sides of the base and height of the tank respectively, then find the area A of metal sheet used.
 - (ii) Find the area of metal sheet in terms of x only.
 - (iii) Area of metal sheet used is least then what will be the length of side of base?
- OR
- (iii) Find the least area of metal sheet used.

SECTION-E

(Questions 35 to 38 carry 5 marks each)

35. Differentiate the function $y = x^{\sin x} + (\sin x)^{\cos x}$ with respect to x .
- OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

36. Find A^{-1} , where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. Hence solve the following system of equations using Matrix Method, $4x + 2y + 3z = 2$, $x + y + z = 1$, $3x + y - 2z = 5$.

37. Let $A = \{1, 2, 3, \dots, 9\}$ and let R be the relation defined on $A \times A$ by $(a, b)R(c, d)$ if $a + d = b + c$. Prove that R is an equivalence relation. Also find the equivalence class $[(2, 5)]$.
- OR

Show that the function $f: R \rightarrow R$, defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$ is neither one-one nor onto.

38. Maximize $Z = x + 2y$, subject to the constraints,
 $x + 2y \geq 100$, $2x + y \leq 200$, $2x - y \leq 0$, $x \geq 0$, $y \geq 0$
 Find also the point(s) at which function attains maximum value

****END OF THE QUESTION PAPER****

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INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
SUBJECT: MATHEMATICS
SUBJECT CODE : 041



CLASS : XII
 DATE: 14-09-2023

TIME ALLOTTED : 3 HRS.
 MAXIMUM MARKS: 80 marks

GENERAL INSTRUCTIONS:

1. This question paper consists of **five sections A, B, C, D and E**. Each section is compulsory.
2. **Section A** comprises of 18 MCQs of one mark each (from Q01-Q18) and Assertion-Reasoning based questions (from Q 19- Q20)
3. **Section B** comprises of 05 Very Short Answer (VSA)-type questions of 2 marks each (from Q21-Q25) .
4. **Section C** comprises of 06 Short Answer (SA)-type questions of 3 marks each (from Q26-Q31) .
5. **Section D** comprises of 03 Case-study based questions (from Q32-Q34)
6. **Section E** comprises of 04 Long Answer (LA)-type questions of 5 marks each (from Q35-Q38)
7. There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

SECTION-A
(Multiple Choice Questions)
Question numbers 1 to 20 carry 1 mark each

1. The function $f(x) = x^x$ has a stationary point at x equals to
 (a) e (b) $\frac{1}{e}$ (c) 1 (d) \sqrt{e}
2. The function $f(x) = \tan x - x$
 (a) always increases (b) always decreases
 (c) never increases (d) neither increasing nor decreasing
3. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?
 (a) $f(x) + g(x)$ (b) $f(x) - g(x)$ (c) $f(x) \cdot g(x)$ (d) $\frac{g(x)}{f(x)}$
4. If $y = \log_7(\log x)$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{1}{x \log x \log 7}$ (b) $\frac{-1}{x \log x \log 7}$ (c) $\frac{1}{x \log x}$ (d) none of these
5. The set of points, where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable, is
 (a) R (b) $R - \left\{\frac{1}{2}\right\}$ (c) $(0, \infty)$ (d) none of these
6. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i + 3j, i < j \\ 5, i = j \\ 3i - 2j, i > j \end{cases}$$

The number of elements in A which are more than 5, is

- (a) 3 (b) 4 (c) 5 (d) 6

7. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then depth of the wheat is increasing at the rate of

- (a) 1 m/h (b) 0.1 m/h (c) 1.1 m/h (d) 0.5 m/h

8. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + p, x < 4 \\ p + q, x = 4 \\ \frac{x-4}{|x-4|} + q, x > 4 \end{cases}$ Then $f(x)$ is continuous at $x = 4$ when

- (a) $p = 0, q = 0$ (b) $p = 1, q = 1$ (c) $p = -1, q = 1$ (d) $p = 1, q = -1$

9. Set A has 3 elements and set B has 4 elements. Then, the number of injective mappings that can be defined from A to B is

- (a) 144 (b) 12 (c) 24 (d) 64

10. If $\tan^{-1}x = y$, then

- (a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$ (c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

11. If A and B are square matrices of the same order 3, such that $|A|=3$ and $AB = 2I$, then the value of $|B|$ is

- (a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) 1

12. The value of θ in $[0, 2\pi]$ such that the matrix $\begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\pi + \theta) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\pi - \theta) & \tan(\pi - \theta) & 0 \end{bmatrix}$ is skew symmetric, is

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

13. The probability of solving the specific problems independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then the probability that exactly one of them solves the problem is

- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

14. If two events are independent, then

- (a) They must be mutually exclusive (b) sum of their probabilities must be equal to 1
(c) (a) and (b) both are correct (d) none of the above is correct

15. The region represented by the inequalities $x, y \geq 0, y \leq 6, x + y \leq 3$ is

- (a) Unbounded in the first quadrant (b) unbounded in the first and second quadrant
(c) bounded in the first quadrant (d) none of these

16. In an LPP, if the objective function $z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is
 (a) 0 (b) 2 (c) finite (d) infinite
17. A point 'c' in the domain of f is called critical point of 'f', if
 I $f'(c) = 0$.
 II f is not differentiable at c.
 Choose the correct option
 (a) Either I or II are true
 (b) Only I is true
 (c) Only II is true
 (d) Neither I nor II is true
18. If $y = \log \sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by
 (a) ∞ (b) 1 (c) 0 (d) $\frac{1}{2}$

Following are Assertion-Reasoning based questions (from Q19-Q20):

Read the following statements carefully to mark the correct option out of the options given below.

- (a) Both **A** and **R** are true and **R** is the correct explanation of **A**.
 (b) Both **A** and **R** are true but **R** is not the correct explanation of **A**.
 (c) **A** is true but **R** is false.
 (d) **A** is false but **R** is true.
19. **Assertion (A):** The function $f: \{1,2,3,4\} \rightarrow \{x,y,z,p\}$ defined by
 $f = \{(1,x), (2,y), (3,z), (4,z)\}$ is a bijective function
Reason (R): The function $f: \{1,2,3\} \rightarrow \{x,y,z,p\}$ such that $f = \{(1,x), (2,y), (3,z)\}$ is one-one.
20. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then
Assertion (A): $f(x)$ has a minimum at $x=1$
Reason (R): When $\frac{d}{dx}(f(x)) < 0 \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0 \forall x \in (a, a+h)$; where 'h' is an infinitely small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

SECTION-B

(Questions 21 to 25 carry 2 marks each)

21. Differentiate $\sqrt{\sec \sqrt{x}}$ with respect to x.
22. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value
23. If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.
 OR
 Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.
24. For what values of x and y are the following matrices equal?

If $A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix}$ and $B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

25. Find the value of $\sin^{-1}[\cos(\frac{3\pi}{5})]$.

OR

Find the domain of the function $\cos^{-1}(\sqrt{x-1})$.

SECTION-C

(Questions 26 to 31 carry 3 marks each)

26. A and B appear for an interview for two posts. The probability of A's selection is $\frac{1}{3}$ and that of B's selection is $\frac{2}{5}$. Find the probability that

- (i) only one of them is selected.
- (ii) at least one of them is selected.

27. Let $A = \{0,1,2,3\}$ and define a relation R on A as $R = \{(0,0)(0,1)(0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$. Is R reflexive, symmetric and transitive? Give reasons to the answer.

28. Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even.

Find whether the function f is one-one, onto or bijective.

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of number of aces.

OR

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred at random from bag I to bag II and then a ball is drawn from bag II. Find the probability that ball drawn is red

30. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = O$. Hence deduce A^{-1} .

OR

Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrices.

31. Find the intervals in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.

SECTION-D

(This section comprises of 3 Case study/ passage based questions of 4 marks each with 3 sub-parts (i), (ii), (iii) of marks 1,1,2 respectively.)

32. In answering a question on a multiple choice test for class XII , a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, A be the events that the student knows the answer , guesses the answer and answers correctly respectively.

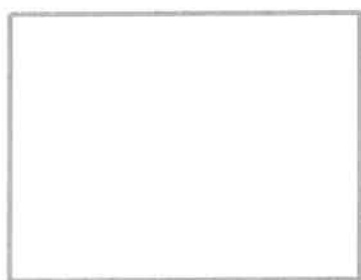


- (i) Find the probability that student answered correctly.
- (ii) Find $\sum_{i=1}^2 P(E_i|A)$
- (iii) Find the probability that the student knows the answer given that he answered it correctly.

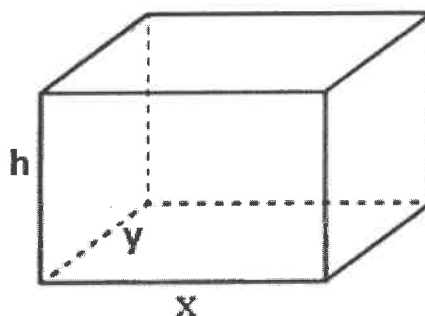
OR

- (iii) Find the probability that the student guesses the answer given that he answered it

33.



metal sheet



To conserve water for emergency needs in a village, the gram panchayat decides to construct an open tank with a square base from a sheet of metal which has been donated by a company. From the account of social welfare account. The area of metal sheet donated is to be least and water to be stored is 4000 m^3

- (i) If x, y and h represent the sides of the base and height of the tank respectively, then find the area A of metal sheet used.
- (ii) Find the area of metal sheet in terms of x only.
- (iii) Area of metal sheet used is least then what will be the length of side of base?

OR

(iii) Find the least area of metal sheet used.

34. A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below.

| Market | Products(in numbers) | | |
|--------|----------------------|--------|-----------|
| | Pencil | Eraser | Sharpener |
| A | 10,000 | 2000 | 18,000 |
| B | 6000 | 20,000 | 8000 |

If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then based on the above information answer the following using matrix algebra :

- (i) Express the total revenue as product of two matrices.
- (ii) Express the total cost as product of two matrices
- (iii) Find the profit in market A.

OR

- (iii) Find the profit in market B.

SECTION-E

(Questions 35 to 38 carry 5 marks each)

35. Find A^{-1} , where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. Hence solve the following system of equations using Matrix Method, $4x + 2y + 3z = 2$, $x + y + z = 1$, $3x + y - 2z = 5$.

36. Show that the relation R defined by $(a, b)R(c, d)$ iff $a + d = b + c$ on $A \times A$, where $A = \{1, 2, 3, \dots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]$; $a, b, c, d \in A$.

OR

Show that the function $f: R \rightarrow R$, defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$ is neither one-one nor onto.

37. Differentiate the function $y = x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

38. Maximize $Z = 5x + 10y$, subject to the constraints,
 $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$
 Find also the point(s) at which function attains maximum value

END OF THE QUESTION PAPER