SET	A/B/C

INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
MATHEMATICS (041)

CLASS: XII   Max.Marks: 80

MARKING SCHEME-VALUE POINTS					
SET	Q.NO	SET A	SET B	SET C	MARKS SPLIT UP
A	1.	(c) 24	b) 1	(b) $\frac{1}{e}$	1
	2.	(d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$	d) p = 1 , q = -1	(a) always increases	1
	3.	a)) $\frac{8}{3}$	(b) $\frac{1}{e}$	(d) $\frac{g(x)}{f(x)}$	1
	4.	d)) $\frac{\pi}{6}$	(a) always increases	(a) $\frac{1}{x\ logx\ log7}$	1
	5.	(b) $\frac{1}{2}$	(b) 4	(b) $R - \left\{\frac{1}{2}\right\}$	1
	6.	(d) none of the above is correct	(a) $1\ m/h$	(b) 4	1
	7.	(c) bounded in the first quadrant	(a) $\frac{1}{x\ logx\ log7}$	(a) $1\ m/h$	1
	8.	(d) infinite	(b) $R - \left\{\frac{1}{2}\right\}$	d) p = 1 , q = -1	1
	9.	(a) Either I or II are true	(d) $\frac{g(x)}{f(x)}$	(c) 24	1
	10.	b) 1	(c) 24	(d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$	1
	11.	d) p = 1 , q = -1	(d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$	a)) $\frac{8}{3}$	1
	12.	(b) $\frac{1}{e}$	a)) $\frac{8}{3}$	d)) $\frac{\pi}{6}$	1
	13.	(a) always increases	d)) $\frac{\pi}{6}$	(b) $\frac{1}{2}$	1
	14.	(d) $\frac{g(x)}{f(x)}$	(b) $\frac{1}{2}$	(d) none of the above is correct	1
	15.	(b) $R - \left\{\frac{1}{2}\right\}$	(d) none of the above is correct	(c) bounded in the first quadrant	1
	16.	(a) $1\ m/h$	(c) bounded in the first quadrant	(d) infinite	1

	17.	(a) $\frac{1}{x\, logx\, log7}$	(d) infinite	(a) Either I or II are true	1
	18.	(b) 4	(a) Either I or II are true	b) 1	1
	19.	(d) **A** is false but **R** is true.	a) Both **A** and **R** are true and **R** is the correct explanation of **A.**	(d) **A** is false but **R** is true.	1
	20.	a) Both **A** and **R** are true and **R** is the correct explanation of **A.**	d) **A** is false but **R** is true	a) Both **A** and **R** are true and **R** is the correct explanation of **A.**	1
	21.	**SECTION : A** $sin^{-1}\left[cos\left(\frac{3\pi}{5}\right)\right] = \, = sin^{-1}\left[sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right] = sin^{-1}\left[sin\left(-\frac{\pi}{10}\right)\right] = \frac{-\pi}{10}$ For f to be defined $\Rightarrow\ x-1\ \geq 0\ and -1 \leq\ \sqrt{(x-1)}\ \leq 1$ $\Rightarrow\ x \geq 1\ and\ 0 \leq\ \ x-1 \leq 1$ $\Rightarrow\ x \geq 1\ and\ 1 \leq\ \ x \leq 2$ $x \in [1\,,2\,]$ OR **Apply the chain rule**			1 + 1 1 1 1
	23	$\frac{\mathbf{1}}{\mathbf{4}\sqrt{\boldsymbol{x}}} \times \frac{\boldsymbol{sec^2}\sqrt{\boldsymbol{x}}}{\sqrt{\boldsymbol{tan}\sqrt{\boldsymbol{x}}}}$			1
	24	Let $y = x+\frac{1}{x} \Rightarrow \frac{dy}{dx} = 1-\frac{1}{x^2}$, $\frac{dy}{dx} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$ $\frac{d^2y}{dx^2} = +\frac{2}{x^3}$, therefore $\frac{d^2y}{dx^2}$ (at $x = 1$) > 0 and $\frac{d^2y}{dx^2}$ (at $x = -1$) < 0 Hence local maximum value of y is at x = –1 and the local maximum value = – 2. Local minimum value of y is at x = 1 and local minimum value = 2. Therefore, local maximum value (–2) is less than local minimum value 2.			$\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}$

It is given that, A=B

$$\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Since, matrices are equal then, their corresponding elements are also equal.

⇒ 2x+1=x+3 ------ (1)

⇒ 2y=y_2+2 $\frac{1}{2}$

⇒ y_2−5y=−6 ------ (2)

From (1),

⇒ 2x+1=x+3

⇒ x=2

From (2), $1\frac{1}{2}$

⇒ y_2−5y=−6

⇒ y_2−5y+6=0

⇒ y_2−3y−2y+6=0

⇒ y(y−3)−2(y−3)=0

⇒ (y−3)(y−2)=0

⇒ y=3 or y=2

∴ We get x=2 and y=3 or y=2

25.

Given

y=3e^{2x}+2e^{3x} ....(i)

Differentiating w.r.t. x

$$\frac{dy}{dx} = 3.2e^{2x} + 2.3e^{3x} = 6e^{2x} + 6e^{3x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 6e^{2x} + \frac{6(y-3e^{2x})}{2} \qquad \text{(using (i))}$$

$$\Rightarrow \quad \frac{dy}{dx} = 6e^{2x} + 3y - 9e^{2x} = -3e^{2x} + 3y \quad \ldots (ii)$$ $\frac{1}{2}$

Differentiating again w.r.t. x $\frac{1}{2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3.\frac{dy}{dx} - 6e^{2x} \qquad \ldots (iii)$$

From (ii) $\frac{dy}{dx} - 3y = -3e^{2x}$

$$\Rightarrow \quad \frac{\frac{dy}{dx} - 3y}{-3} = e^{2x}$$ $\frac{1}{2}$

Substitute in (iii)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3.\frac{dy}{dx} - 6\left(\frac{\frac{dy}{dx} - 3y}{-3}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 2\frac{dy}{dx} - 6y$$ $\frac{1}{2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0$$

OR

Let u = $\sin^2 x$ and v = $e^{\cos x}$

Differentiating u and v w.r.t. x, we get

du/dx = 2sinx(d/dx)(sinx) = 2sinxcosx $\frac{1}{2}$

and dv/dx =$e^{\cos x}$(d/dx(sinx) = $e^{\cos x}$(-sinx) = (-sinx)$e^{\cos x}$ $\frac{1}{2}$

$$\text{Now, } \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x\cos x}{(-\sin x)e^{\cos x}} = -\frac{2\cos x}{e^{\cos x}}$$ 1

SECTION : C

f(n)={2n+1,if n is odd2n,if n is even} for all n∈N
f:N→N is defined as
It can be observed that:

f(1) =1 and f(2) =1 1
∴f(1)=f(2), where 1≠2
∴f is not one-one.

Consider a natural number (n) in co-domain N
Case I: n is odd
∴n=2r+1 for some r∈N. 1
Then, there exists 4r+1∈N such that f (4r+1) = 2r+1

	26	Case II: n is even ∴ n=2r for some r ∈N. Then, there exists 4r∈N such that f (4r) = 2r ∴ f is onto. Hence, f is not a bijective function	 1
		OR	
		$f\ (x) = \dfrac{x-2}{x-3}$ f(x1)=f(x2) ⇒ x1=x2 So, f(x) is one-one	1
		f(x) = (x−3) / (x−2) y = (x−3) / (x−2) y(x−3) = x−2 yx−3y = x−2 yx−x = 3y−2 x(y−1) =3y−2 x = (3y−2) / (y−1)	 1
		Show that f(x)=y f(x) is onto. So f(x) is bijective	
	27.	Here A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $\therefore\ A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$	
		$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	1
		$\therefore\ A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$	

Thus $A^2 - 5A + 7I = 0$ 1

Pre-multiplying by A^{-1} on both sides, we get

$A^{-1}(A^2 - 5A + 7I) = A^{-1}.0$

$A^{-1}A^2 - 5A^{-1}A + 7A^{-1}I = 0$

$A - 5I + 7A^{-1} = 0$

$$\Rightarrow A^{-1} = \frac{1}{7}(5I - A) = \frac{1}{7}\left(5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\right)$$

1

$$= \frac{1}{7}\left(\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix} - \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\right)$$

$$= \frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

OR

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$$A = \begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$$

then $$A' = \begin{bmatrix}2 & 7 & 1\\ 4 & 3 & -2\\ -6 & 5 & 4\end{bmatrix}$$ 1

Let $$P = \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\begin{bmatrix}4 & 11 & -5\\ 11 & 6 & 3\\ -5 & 3 & 8\end{bmatrix}$$

$$= \begin{bmatrix}2 & \frac{11}{2} & -\frac{5}{2}\\ \frac{11}{2} & 3 & \frac{3}{2}\\ -\frac{5}{2} & \frac{3}{2} & 4\end{bmatrix}$$

Since $P' = P$ 1

		Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix}$ $= \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$ 1 Since $Q' = -Q$ $\therefore Q$ is a skew symmetric matrix.	1
	28	$f'(x) = 6x^2 - 6x - 36$ $= 6(x^2 - x - 6)$ $= 6(x - 3)(x + 2)$ $f'(x) = 0$ or $x = -2, x = 3$ $\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ getting $f'(x)$ + ve in $(-\infty, -2) \cup (3, \infty)$ and –ve in $(-2, 3)$ $\therefore f(x)$ is strictly increasing in $(-\infty, -2) \cup (3, \infty)$, and strictly decreasing in $(-2, 3)$	1 1 1
	29	Given: A and B appear for an interview for two posts such that the probability of A's selection is $\frac{1}{3}$ and that of B's selection is $\frac{2}{5}$. Let E = event that A is selected Let F = event that B is selected $\Rightarrow P(E) = \frac{1}{3}$ and $P(F) = \frac{2}{5}$ As we know that, if $P(A) = x$ then $P(\bar{A}) = 1 - x$ $\Rightarrow P(\bar{E}) = 1 - (\frac{1}{3}) = \frac{2}{3}$ and $P(\bar{F}) = 1 - (\frac{2}{5}) = \frac{3}{5}$ $\therefore$ P(event that one of them is selected) $= P(E \cap \bar{F}) + P(\bar{E} \cap F)$	1 1

		As we know that, if A and B are two independent events then P(A ∩ B) = P(A) × P(B) $\Rightarrow P(E \cap \bar{F}) = P(E) \times P(\bar{F}) = (\frac{1}{3}) \times (\frac{3}{5}) = \frac{1}{5}$ $\Rightarrow P(\bar{E} \cap F) = P(\bar{E}) \times P(F) = (\frac{2}{3}) \times (\frac{2}{5}) = \frac{4}{15}$ $\Rightarrow$ P(event that one of them is selected) = $(\frac{1}{5}) + (\frac{4}{15}) = \frac{7}{15}$ (ii) P(at least one of them is selected) = 1- P(none of them selected) = 1-P$\overline{(E} \cap \bar{F})$ =1-$(\frac{2}{3} X \frac{3}{5})$ =$\frac{3}{5}$	$1\frac{1}{2}$ $1\frac{1}{2}$
	30	Since A = {0, 1, 2, 3} R : A → A Since, 0, 1, 2, 3 ∈ A and (0, 0), (1, 1), (2, 2) (3, 3) ∈ R Hence, for each a ∈ A (a, a) ∈ R ∴ R is a reflexive relation. Since, (0, 1) ∈ R Then (1, 0) ∈ R (0, 3) ∈ R Then (3, 0) ∈ R Hence, if (a, b) ∈ R Then (b, a) ∈ R ∴ Relation R is symmetric relation. Since, (1, 0) ∈ R, (0, 3) ∈ R but (1, 3) ∉ R ∴ Relation R is not transitive.	1 1 1
	31	Let X denote the number of kings in a draw of two cards. X is a random variable which can assume the values 0, 1 or 2. Now P(X = 0) = P (no king)	

Now P(X = 0) = P (no king)

$$= \frac{^{48}C_2}{^{52}C_2} = \frac{\dfrac{48!}{2!(48-2)!}}{\dfrac{52!}{2!(52-2)!}} = \frac{48\times 47}{52\times 52} = \frac{188}{221}$$ $1\frac{1}{2}$

$1\frac{1}{2}$

Thus, the probability distribution of X is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\frac{1}{2}$

Now Mean of X = E(X) = $\sum_{i=1}^{n} x_i p(x_i)$ $\frac{1}{2}$

$$= 0\times\frac{188}{221} + 1\times\frac{32}{221} + 2\times\frac{1}{221} = \frac{34}{221}$$ $\frac{1}{2}$

$\frac{1}{2}$

OR

E1=Ball transferred from Bag I to Bag II is red

E_2=Ball transferred from Bag I to Bag II is black

A=Ball drawn from Bag II is red in colour

$P(E_1)=3/7$

$P(E_2)=4/7$ $\frac{1}{2}$ for each step

$P(A/E_1)=5/10=1/2$

$P(A/E_2) = 4/10=2/5$

Required probability = P(A)

1

$= P(E_1)P(A/E_1)+P(E_2)P(A/E_2)$

$= \frac{31}{70}$

SECTION : D

Let the sales of Pencil, Eraser and Sharpener be denoted by matrix X

$$X = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} \text{Pencil} & \text{Eraser} & \text{Sharpener} \end{array}$$
$$X = \begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix} \begin{array}{l} \text{Market A} \\ \text{Market B} \end{array}$$

1

	32	Let the unit sale price of Pencil, Eraser and Sharpener be denoted by matrix Y ***Unit sale price*** $\text{Let } Y = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \begin{array}{l} \textbf{Pencil} \\ \textbf{Eraser} \\ \textbf{Sharpener} \end{array}$ (i) Total Revenue = Total sales x Unit sales price = XY $= \begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}_{3\times 1}$ (ii) Let the unit cost price of Pencil, Eraser and Sharpener be denoted by matrix Z ***Unit cost price*** $\text{Let } Z = \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \begin{array}{l} \textbf{Pencil} \\ \textbf{Eraser} \\ \textbf{Sharpener} \end{array}$ Now, Total Cost = Total sales x Unit cost price = XZ $= \begin{bmatrix} 10{,}000 & 2000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}_{3\times 1}$	1 1

Total Revenue = Total sales x Unit sales price | 2

= XY

$$= \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}_{2\times3} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 10,000(2.50) + 2,000(1.50) + 18,000(1) \\ 6,000(2.50) + 20,000(1.50) + 8,000(1) \end{bmatrix}$$

$$= \begin{bmatrix} 25,000 + 3,000 + 18,000 \\ 15,000 + 30,000 + 8,000 \end{bmatrix} = \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix}$$

$$\textbf{Total Revenue} = \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} \begin{array}{l} \longrightarrow \textbf{Market A} \\ \longrightarrow \textbf{Market B} \end{array}$$

Hence,

Total revenue of Market A = Rs. 46,000

Total revenue of Market B = Rs. 53,000

Total Cost = Total sales x Unit cost price

= XZ

$$= \begin{bmatrix} 10,000 & 2000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}_{2\times3} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 10,000(2.00) + 2000(1.00) + 18,000(0.50) \\ 6,000(2.00) + 20,000(1.00) + 8,000(0.50) \end{bmatrix}$$

$$= \begin{bmatrix} 20,000 + 2,000 + 9,000 \\ 12,000 + 20,000 + 4,000 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{31,000} \\ \mathbf{36,000} \end{bmatrix} \begin{array}{l} \longrightarrow \textbf{Market A} \\ \longrightarrow \textbf{Market B} \end{array}$$

∴ Total cost of Market A = Rs. 31,000

Profit = Revenue – Cost

$$= \begin{bmatrix} 46{,}000 \\ 53{,}000 \end{bmatrix} - \begin{bmatrix} 31{,}000 \\ 36{,}000 \end{bmatrix}$$

$$= \begin{bmatrix} 15{,}000 \\ 17{,}000 \end{bmatrix}$$

Thus,

Profit in Market A = Rs. 15,000

Profit in Market B = Rs. 17,000

33 Let E1 and E2 be the respective events that the student knows the answer and he guesses the answer.

Let A be the event that the answer is correct.

$$P(E_1) = \frac{3}{5}, \qquad P(E_2) = \frac{2}{5}$$

P(A|E1)=1

$$P(A|E_2) = \frac{2}{5}$$

(i) P(A)= $P(E_1)$.P(A | E1)+ $P(E_2).P(A|E_2) = \frac{11}{15}$ 1

(ii) $\sum_{i=1}^{2} P(E_i|A)$ =P(E1 | A) + P(E2 | A) = $\frac{9}{11} + \frac{2}{11} = 1$ 1

(iii) By Baye's theorem,

$$P(E_1 | A) = \frac{9}{11}$$ 2

OR

$$P(E_2 | A) = \frac{2}{11}$$

34 **As base is a square and tank is open at the top. Hence $x = y$.**

(*i*) as area of metal sheet A = area of base + area of four sides = $x \times x + 4x \times h$ 1

$$A = x^2 + 4xh$$

(*ii*) as we eliminate h, $h = \frac{4000}{x^2}$ 1

$$\therefore \quad A = x^2 + 4x \cdot \frac{4000}{x^2} = x^2 + \frac{16000}{x}$$

34.	(*iii*) For least area $\frac{dA}{dx} = 0 \Rightarrow 2x - \frac{16000}{x^2} = 0$ $\Rightarrow x^3 = 8000 \Rightarrow x = 20$ m $\frac{d^2A}{dx^2} > 0$ $\frac{d^2A}{dx^2} = 2 + \frac{32000}{x^3} > 0$ for $x = 20$ **OR**	2
35	(*iii*) 1200 m^2	

SECTION : E

R is an equivalance relation if R is reflexive, symmetric and transitive.

a)checking if it is reflexive; — 1

Given R in A×A and (a,b) R(c,d)such that a+d=b+c

For reflexive, consider (a,b) R (a,b)(a,b)∈A

and applying given condition ⇒ a+b = b+a; which is true for all A

∴R is reflexive.

b)checking if it is symmetric;

given(a,b) R (c,d)such that a+d = b+c

consider (c,d) R (a,b)on A×A

applying given condition ⇒ c+b = d+a which satisfies given condition — 1

Hence R is symmetric.

c)checking if it is transitive;

Let (a,b) R(c,d) and (c,d) R (e,f)

And (a,b), (c,d), (e,f) ∈A×A

applying given condition:⇒ a + d = b + c →1 and c + f = d + e →2

equation 1 ⇒ a−c = b−d

now add equation1and2;

⇒ a − c + c + f = b−d + d + e

⇒ a + f = b + e — 2

∴(a,b) R(e,f) also satisfies the condition

Hence R is transitive.

Equivalence class [3,4] =

{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9),(9,10)} — 1

OR

For a function to be one to one, if we assume $f(x_1) = f(x_2)$, then $x_1 = x_2$

Given, f: R→R defined by $f(x) = \frac{x}{x^2+1}$, ∀x∈R

Thus for one-one function, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$ $2\frac{1}{2}$

$\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_2x_1 = 1$, if $x_2 \neq x_1$

⇒ f is not one-one function.

Also, a function is onto if and only if for every y in the co-domain, there is x in the domain such that f(x) = y $2\frac{1}{2}$

Let f(x) = y

$\Rightarrow \frac{x}{x^2+1} = y$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

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Now, substituting this x in f(x) = y we can see that this function is not onto.

Let $u = x^{\sin x}$ and $v = \sin x^{\cos x}$

$\Rightarrow y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 2

Consider $u = x^{\sin x}$

$\log u = \sin x \log x$

$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} + \log x(\cos x)$

$\Rightarrow \frac{du}{dx} = u\left(\frac{\sin x}{x} + \cos x \log x\right)$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$

Consider $v = (\sin x)^{\cos x}$

$\log v = \cos x \log \sin x$

$$\frac{1}{v}\frac{dv}{dx} = \frac{\cos x}{\sin x}(\cos x) + \log \sin x(-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right)$$ 2

$$\therefore \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) + (\sin x)^{\cos x}\left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right)$$ 1

Given,

$x = a(\cos t + t \sin t)$

$$\frac{dx}{dt} = a\,(-\sin t + \sin t + t\,.\cos t)$$ 1

$$\frac{dx}{dt} = a\,.t.\cos t$$

Now, $y = a(\sin t - t \cos t)$ $\quad \frac{dy}{dt} = a\,.t.\sin t$ 1

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{a\,.t.\sin t}{a\,.t.\cos t}$$ 1

$$\frac{dy}{dx} = \tan t$$

$$\frac{d^2y}{dx^2} = \sec^2 t \div \frac{dx}{dt}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \div a.t.\cos t$$

Hence $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{a \cdot t}$ $1\frac{1}{2}$

$$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{8\sqrt{2}}{a\pi}$$ $\frac{1}{2}$

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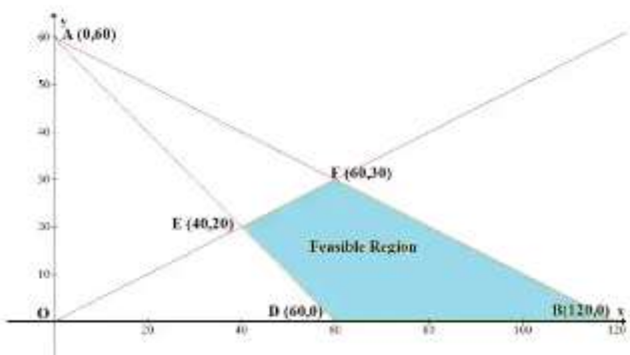
The shaded region BDEF determined by linear inequalities shows the feasible region
Let us evaluate the objective function Z at each point as shown below
At B(120,0), Z=5×120+10×0=600
At D(60,0), Z=5×60+10×0=300
At E(40,20), Z=5×40+10×20=400
At F(60,30), Z=5×60+10×30=600
Hence, Maximum value of Z is 600 at F(60,30)

2



3

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Given

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$$

$$\Rightarrow |A| = 4(-2-1) - 2(-2-3) + 3(1-3)$$
$$= -12 + 10 - 6 = -8 \neq 0$$

A^{-1} exists and we know that $A^{-1} = (1/|A|)\text{adj}.A$

$$A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3,\ A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = 5,\ A_{13} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2$$ 1

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = 7,\ A_{22} = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} = -17,\ A_{23} = -\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = 2$$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1,\ A_{32} = -\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} = -1,\ A_{33} = \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = 2$$

$$\text{adj } A = \begin{bmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|} \text{adj. } A = -\frac{1}{8} \begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} 3 & -7 & 1 \\ -5 & 17 & 1 \\ 2 & -2 & -2 \end{bmatrix}$$

The given system of equation is: $4x + 2y + 3z = 2$

$x + y + z = 1$

$3x + y - 2z = 5$

Hence, this system can be written as AX = B

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$ 2

As $|A| \neq 0$, the given system has a unique solution $X = A^{-1} B$

$$\Rightarrow\quad X = \frac{1}{8} \begin{bmatrix} 3 & -7 & 1 \\ -5 & 17 & 1 \\ 2 & -2 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \qquad \text{[using } (i)\text{]}$$

$$\Rightarrow\quad X = \frac{1}{8} \begin{bmatrix} 6-7+5 \\ -10+17+5 \\ 4-2-10 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 4 \\ 12 \\ -8 \end{bmatrix}$$

		$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \quad \left[\because X = \begin{bmatrix} x \\ y \\ x \end{bmatrix}\right]$ $\Rightarrow \quad x = \frac{1}{2}, y = \frac{3}{2}, z = -1$	2
		SET: B	
	25	$\frac{-1}{4\sqrt{x}} \times \frac{cosec^2\sqrt{x}}{\sqrt{cot\sqrt{x}}}$	1 + 1
	31 37	$f(x) = x^3 - 12x^2 + 36x + 17$ a. For $f(x)$ to be increasing, we must have $f'(x) > 0$ $\Rightarrow 3(x-6)(x-2) > 0$ $\Rightarrow x < 2$ or $x > 6$ $\Rightarrow x \in (-\infty, 2) \cup (6, \infty)$ So, $f(x)$ is increasing on $(-\infty, 2) \cup (6, \infty)$ b. For $f(x)$ to be decreasing, we must have $f'(x) < 0$ $\Rightarrow 3(x-2)(x-6) < 0$ $\Rightarrow 2 < x < 6$ So, $f(x)$ is decreasing on $(2, 6)$. Let A = {1,2,3, ...,9) $\forall (a, b) \in A \times A$ $a + b = b + a$ $\therefore (a, b) R (a, b)$ $\therefore R$ is reflexive. For $(a, b), (c, d) \in A \times A$ $(a, b) R (c, d)$ i.e., $a + d = b + c$ Or $\quad c + b = d + a$ then $(c, d) R (a, b)$ $\therefore R$ is symmetric.	$1\frac{1}{2}$ $1\frac{1}{2}$ 1 1

For $(a, b), (c, d), (e, f) \in A \times A$

If $(a, b)\ R\ (c, d)$ & $(c, d)\ R\ (e, f)$

i.e. $a + d = b + c$ & $c + f = d + e$

Adding, $(a + d) + (c + f) = (b + c) + (d + e)$

Or $a + f = b + e$

then, $(a, b)\ R\ (e, f)$

$\therefore$ R is transitive.

$\therefore$ R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation. 2

$[(2, 5)] = \{(1, 4), (2, 5) (3, 6), (4, 7), (5, 8), (6, 9)\}$ 1

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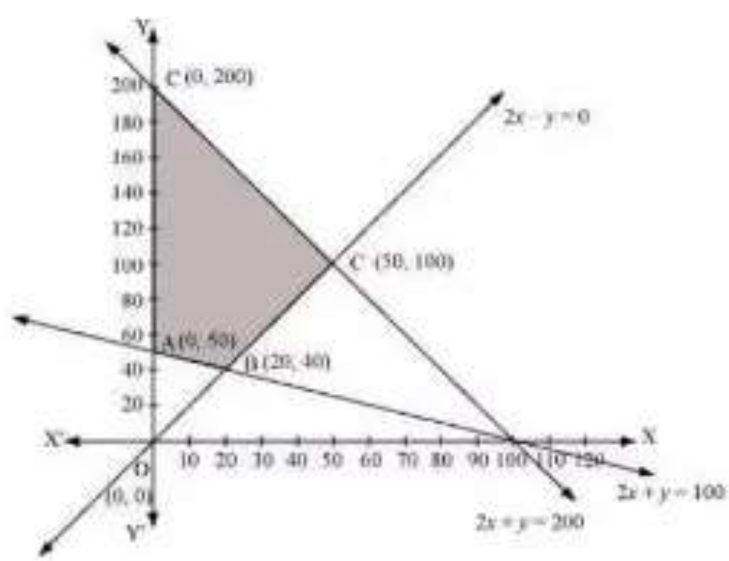
The feasible region determined by the constraints $x + 2y \geq 100$, $2x - y \leq 0$, $2x$ $y \leq 200$; $x \geq 0$, $y \geq 0$ is as shown.

The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200)

The values of Z at these corner points are as follows.

Corner point	$Z = x + 2y$	
A(0, 50)	100	$\rightarrow$ Minimum
B(20, 40)	100	$\rightarrow$ Minimum
C(50, 100)	250	
D(0, 200)	400	$\rightarrow$ Maximum

2



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		The maximum value of Z is 400 at (0, 200)	
		SET: C	
	21	$\frac{1}{4\sqrt{x}}\sqrt{sec\sqrt{x}}\ tan\sqrt{x}$	1 + 1
	28	It is given that: f: W → W is defined as $f(n) = \begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$ For one - one Let f(n) = f(m). It can be observed that if n is odd and m is even, then we will have n − 1 = m + 1 ⇒ n − m = 2 However, this is impossible. Similarly, the possibility of n being even and m being odd can also be ignored under a similar argument. ∴ Both n and m must be either odd or even. Now, if both n and m are odd, Then, we have $f(n) = f(m)$ $\Rightarrow n - 1 = m - 1$ $\Rightarrow n = m$	1
		Again, if both n and m are even, Then, we have $f(n) = f(m)$ $\Rightarrow n + 1 = m + 1$ $\Rightarrow n = m$ ∴ f is one – one.	1
		For onto It is clear that any odd number $2r + 1$ in co-domain **N** is the image of $2r$ in domain **N** and any even number $2r$ in co-domain **N** is the image of $2r + 1$ in domain **N**. ∴ f is onto.	1