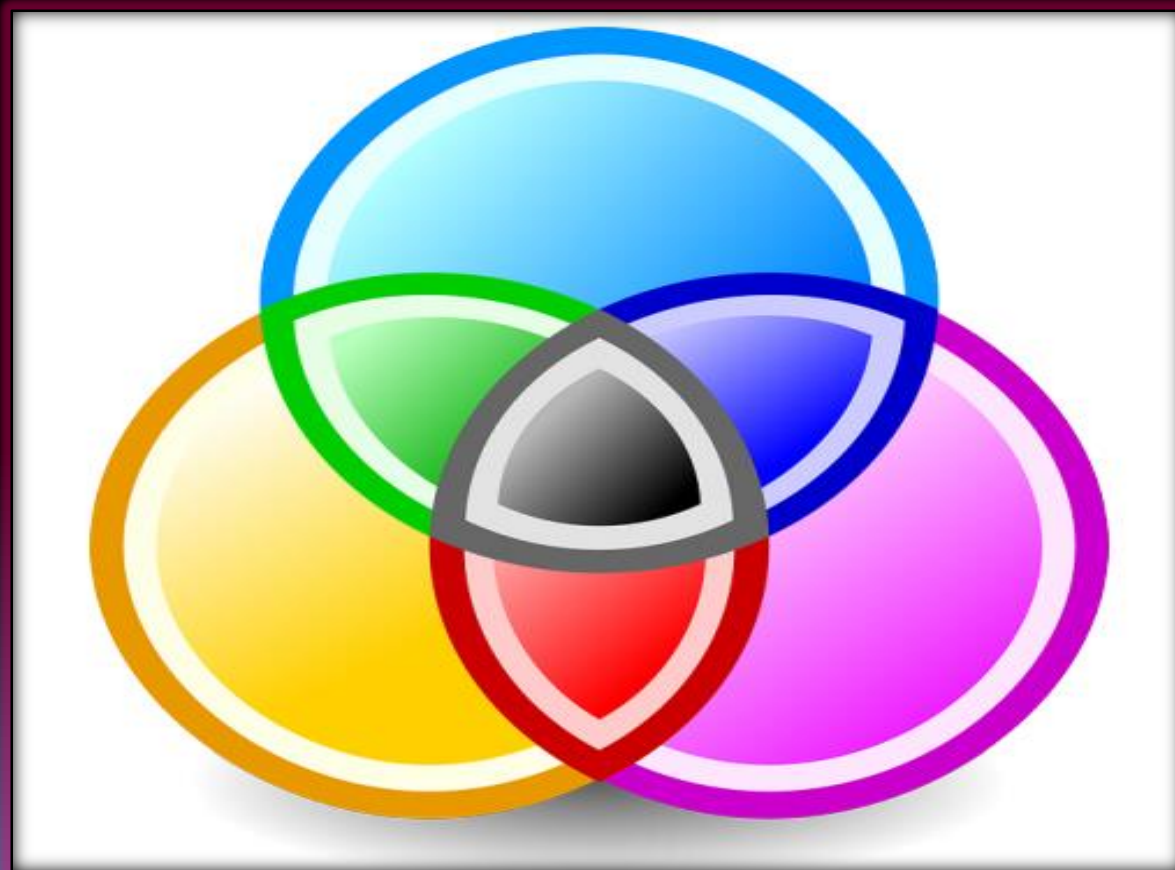




SETS

MODULE 4



QUICK RECAP

- A **SET** is a well defined collection of objects.
- $B = \{x : x \text{ is a natural number which divides } 21\}$ ----- **Set Builder Form**
 $B = \{1, 3, 7, 21\}$ ----- **Roster Form**
- $\phi \rightarrow$ **Null set / Empty Set**
- $B = \{1, 3, 7, 21\}$ ---- **Finite Set**
 $C = \{1, 2, 3, \dots\}$ ---- **Infinite Set**
- **Subset**
- **Power Set** ---- Set of all subsets of a set.
- If $n(A) = m$ then $n[P(A)] = 2^m$
- Intervals as subsets of \mathbb{R} : (a, b) , $(a, b]$, $[a, b)$, $[a, b]$
- $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- $A - B = \{x : x \in A \text{ and } x \notin B\}$
- $A^c = \{x : x \in U \text{ and } x \notin A\}$

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- Using **Venn diagrams** for solving word problems.

CAN YOU TELL
THE ANSWERS

???

Q1) If $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$ and $C = \{1, 2, 3, 4\}$, then what is $A \cap (B - C)$?

- (A) $\{2, 4\}$ (B) \emptyset (C) $\{5, 7, 9\}$ (D) $\{1, 3, 6, 8, 10\}$

Q2) On real axis if $A = [1, 5]$ and $B = [3, 9]$, then $A - B$ is

- (a) $(5, 9)$ (b) $(1, 3)$ (c) $[5, 9)$ (d) $[1, 3)$

QUESTION FOR POLL.....

Q3) If $n(A - B) = 10$, $n(B - A) = 23$, $n(A \cup B) = 50$, then $n(A \cap B)$ is_____.

- (a) 7 (b) 17 (c) 27 (d) 33



BASIC RESULT ABOUT CARDINAL NUMBER FOR THREE SETS

If A, B and C are finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

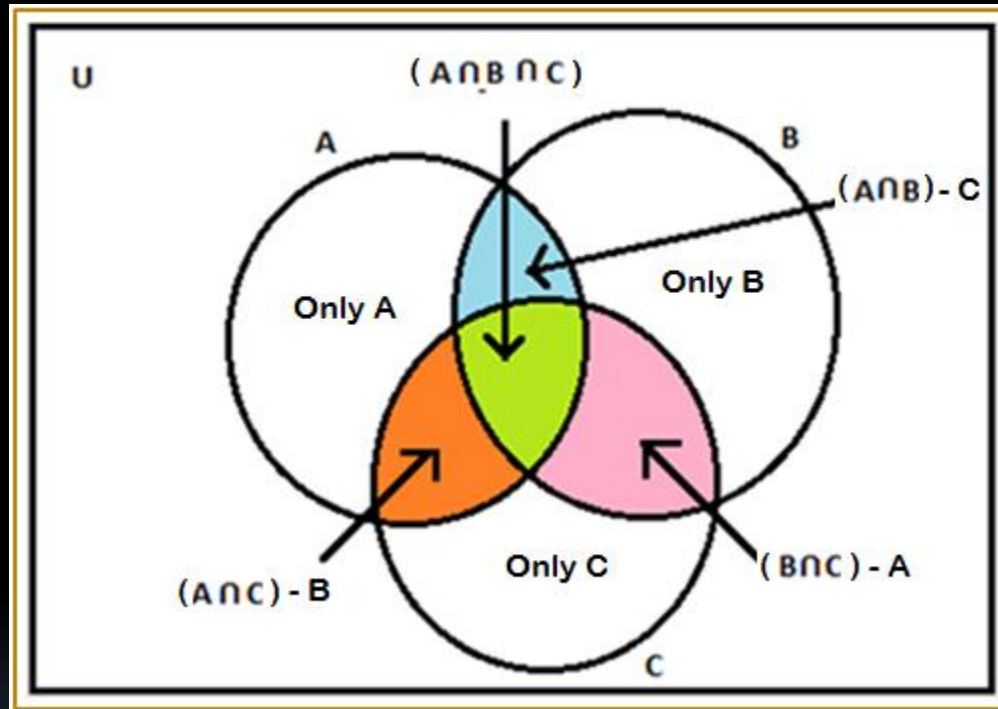
In fact, we have

$$\begin{aligned} n(A \cup B \cup C) &= n[A \cup (B \cup C)] \\ &= n(A) + n(B \cup C) - n[A \cap (B \cup C)] \\ &= n(A) + n(B) + n(C) - n(B \cap C) - n[A \cap (B \cup C)] \\ &= n(A) + n(B) + n(C) - n(B \cap C) - \{n(A \cap B) + n(A \cap C) - n[(A \cap B) \cap (A \cap C)]\} \\ &= n(A) + n(B) + n(C) - n(B \cap C) - \{n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)\} \\ &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \end{aligned}$$

$$A \cap (B \cup C)$$

$$= (A \cap B) \cup (A \cap C)$$

VENN DIAGRAM FOR THREE SETS



The **intersection** of three sets A , B and C is the set of elements that are common to sets A , B and C . It is denoted by $A \cap B \cap C$.

Ten Best Friends

You have a set made up of your ten best friends.

The Universal Set is our Ten Best Friends.

$$U = \{\text{alex, blair, casey, drew, erin, francis, glen, hunter, ira, jade}\}$$

$$\text{Soccer} = \{\text{alex, casey, drew, hunter}\}$$

$$\text{Tennis} = \{\text{casey, drew, jade}\}$$

$$\text{Volleyball} = \{\text{drew, glen, jade}\}$$

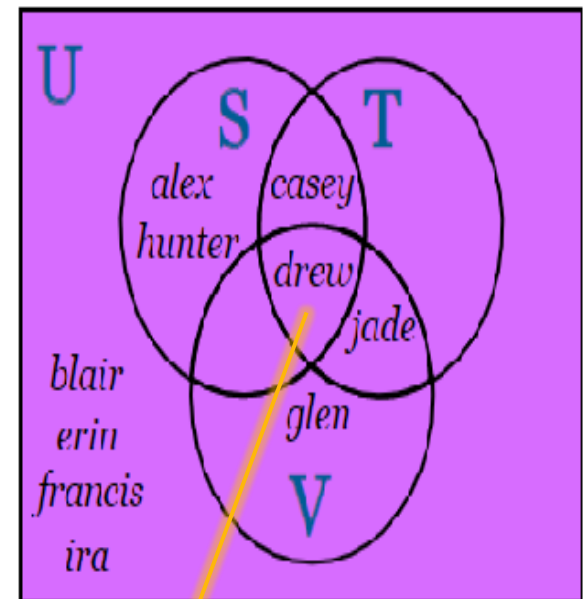
But let's be more "mathematical" and use a Capital Letter for each set:

- **S** means the set of Soccer players
- **T** means the set of Tennis players
- **V** means the set of Volleyball players

You can see that:

- drew plays Soccer, Tennis **and** Volleyball ($S \cap T \cap V$)
- jade plays Tennis and Volleyball ($T \cap V$) - **S**
- alex and hunter play Soccer, but don't play Tennis or Volleyball $S - (T \cup V)$
- no-one plays **only** Tennis $T - (S \cup V)$

The Venn Diagram is now like this:



$S \cap T \cap V$

Example: 1

Draw a Venn diagram to represent the relationship between the sets $X = \{1, 2, 5, 6, 7, 9\}$, $Y = \{1, 3, 4, 5, 6, 8\}$ and $Z = \{3, 5, 6, 7, 8, 10\}$

Solution:

We find that $X \cap Y \cap Z = \{5, 6\}$, $X \cap Y = \{1, 5, 6\}$, $Y \cap Z = \{3, 5, 6, 8\}$ and $X \cap Z = \{5, 6, 7\}$

For the Venn diagram:

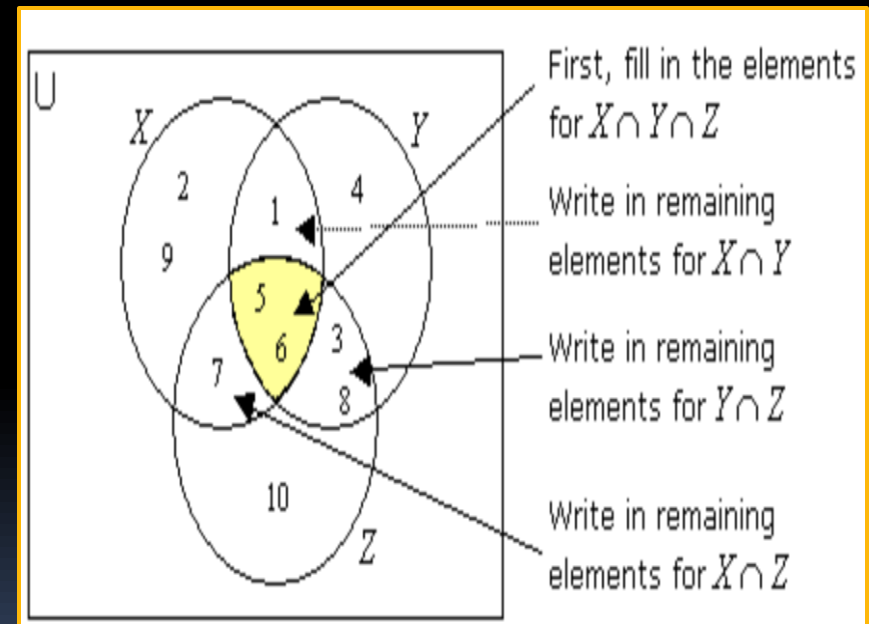
Step 1 : Draw three overlapping circles to represent the three sets.

Step 2 : Write down the elements in the intersection $X \cap Y \cap Z$

Step 3 : Write down the remaining elements in the intersections:

$X \cap Y$, $Y \cap Z$ and $X \cap Z$

Step 4 : Write down the remaining elements in the respective sets.



Again, notice that you start filling the Venn diagram from the elements in the intersection first.

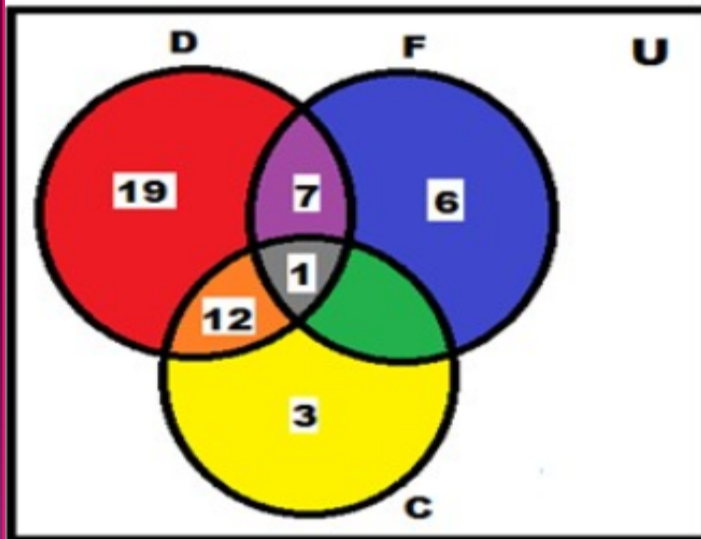
Example: 2

Fifty people are asked about the pets they keep at home. The Venn diagram shows the result.

Let $D = \{ \text{people who have dogs} \}$

$F = \{ \text{people who have fish} \}$

$C = \{ \text{people who have cats} \}$



How many people have

- Dogs?
- Dogs and fish?
- Dogs or cats?
- Fish and cats but not dogs?
- Dogs or fish but not cats?
- All three?
- Neither one of the three?

CAN
YOU
TRY
???

Example 3.

In a survey of university students, 64 had taken mathematics course, 94 had taken chemistry course, 58 had taken physics course, 28 had taken mathematics and physics, 26 had taken mathematics and chemistry, 22 had taken chemistry and physics course, and 14 had taken all the three courses. Find how many had taken one course only.

Solution :

Let M , C , P represent sets of students who had taken mathematics, chemistry and physics respectively

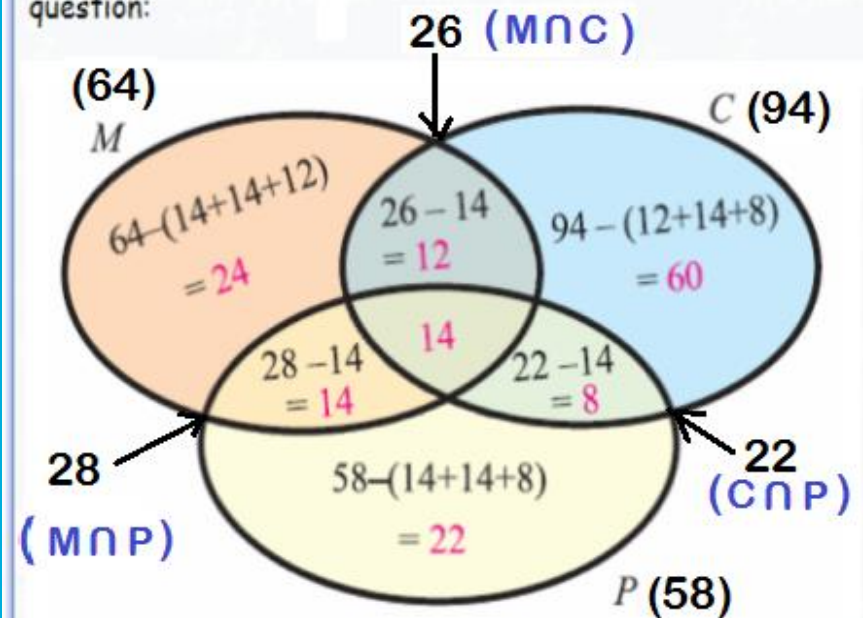
From the given information, we have

$$n(M) = 64, n(C) = 94, n(P) = 58,$$

$$n(M \cap P) = 28, n(M \cap C) = 26, n(C \cap P) = 22$$

$$n(M \cap C \cap P) = 14$$

Venn diagram related to the information given in the question:



From the venn diagram above, we have

$$\text{No. of students who had taken only math} = 24$$

$$\text{No. of students who had taken only chemistry} = 60$$

$$\text{No. of students who had taken only physics} = 22$$

Total no. of students who had taken only one course :

$$= 24 + 60 + 22$$

$$= 106$$

Hence, the total number of students who had taken only one course is 106.

4. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Solution

Let U be the set of car owners investigated,
 M be the set of persons who owned car A
 S be the set of persons who owned car B.

Given that $n(U) = 500$, $n(M) = 400$,
 $n(S) = 200$ $n(S \cap M) = 50$.

Then $n(S \cup M) = n(S) + n(M) - n(S \cap M)$
 $= 200 + 400 - 50$
 $= 550$

But $S \cup M \subset U$ implies $n(S \cup M) \leq n(U)$.

This is a contradiction.

So, the given data is incorrect.

H.W.

Q.1

110 college freshmen were surveyed. 25 took physics, 45 took biology, 45 took mathematics, 10 took physics and mathematics, 8 took biology and mathematics, 6 took physics and biology and 5 took all three.

- How many students took biology, but neither physics nor mathematics?
- How many students took biology, physics or mathematics?
- How many students did not take any of the three subjects?

Q.2

In a survey of 100 students regarding watching T.V., it was found that 28 watch action movies, 30 watch comedy serials, 42 watch news channels, 8 watch action movies and comedy serials, 10 watch action movies and news channels, 5 watch comedy serials and news channels and 3 watch all the three programs. draw a Venn diagram to illustrate this information and find

- how many watch news channels only?
- how many do not watch any of the three programs?

ASSIGNMENT QUESTIONS

1. Describe the set in Roster form [1]

$\{x : x \text{ is a two digit number such that the sum of its digit is } 8\}$

2. Are the following pair of sets equal? Give reasons. [1]

$A = \{x : x \text{ is a letter in the word FOLLOW}\}$

$B = \{y : y \text{ is a letter in the word WOLF}\}$

3. If A and B are two given sets, then $(A - B)$ is sent [1]

Using Venn diagram.

4. $A = \{1, 2, \{3, 4\}, 5\}$ which is incorrect and why. (i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ [1]

5. Fill in the blanks. [1]

(i) $A \cup A' = \dots\dots\dots$ (i) $(A \cup B)' = \dots\dots\dots$

(ii) $(A')' = \dots\dots\dots$ (ii) $(A \cap B)' = \dots\dots\dots$

(iii) $A \cap A' = \dots\dots\dots$

6. List all the element of the set $A = \{x : x \text{ is an integer } x^2 \leq 4\}$ [1]

7. From the sets given below pair the equivalent sets. [1]

$A = \{1, 2, 3\}$, $B = \{x, y, z, t\}$, $C = \{a, b, c\}$, $D = \{0, a\}$

8. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ [1]

Find $(A \cap B) \cap (B \cup C)$

9. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form. [1]

10. Write down all the subsets of the set $\{1, 2, 3\}$ [1]

11. Write down all possible proper subsets of the set $\{1, \{2\}\}$. [1]

12. State whether each of the following statement is true or false. [1]

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint

(ii) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets

13. Write the following as interval [1]

(i) $\{x : x \in \mathbb{R}, -4 < x \leq 6\}$

(ii) $\{x : x \in \mathbb{R}, 3 \leq x \leq 4\}$

CONTINUED...

14. If $U = \{a, e, i, o, u\}$ $A = \{a, e, i\}$ $B = \{e, o, u\}$ And $C = \{a, i, u\}$ [4]

Then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$

15. A and B are two sets such that $n(A - B) = 20 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x + 1$. [4]

Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, Find

(i) the value of x (ii) $n(A \cup B)$

16. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee. [4]

Find (i) how many drink coffee,

(ii) How many drink coffee, but not tea.

17. If A, B, and C, are three sets and U is the universe set such that $n(U) = 1000$, [4]

$n(A) = 300$, $n(B) = 300$ and $n(A \cap B) = 200$ find $n(A' \cap B')$.

18. In a survey of 60 people, it was found that 25 people read news paper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspaper. Find [6]

(i) The no. of people who read at least one of the newspapers.

(ii) The no. of people who read exactly one news paper.

19. There are 20 students in a chemistry class and 30 students in a physics class. Find [6] the number of students which are either in physics class or chemistry class in the following cases.

(i) Two classes meet at the same hour

(ii) The two classes met at different hours and ten students are enrolled in both the courses.

20. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. [6]

Find the no. of students that had taken

(i) only chemistry

(v) mathematics and physics but not chemistry

(ii) only mathematics

(vi) only one of the subjects

(iii) only physics

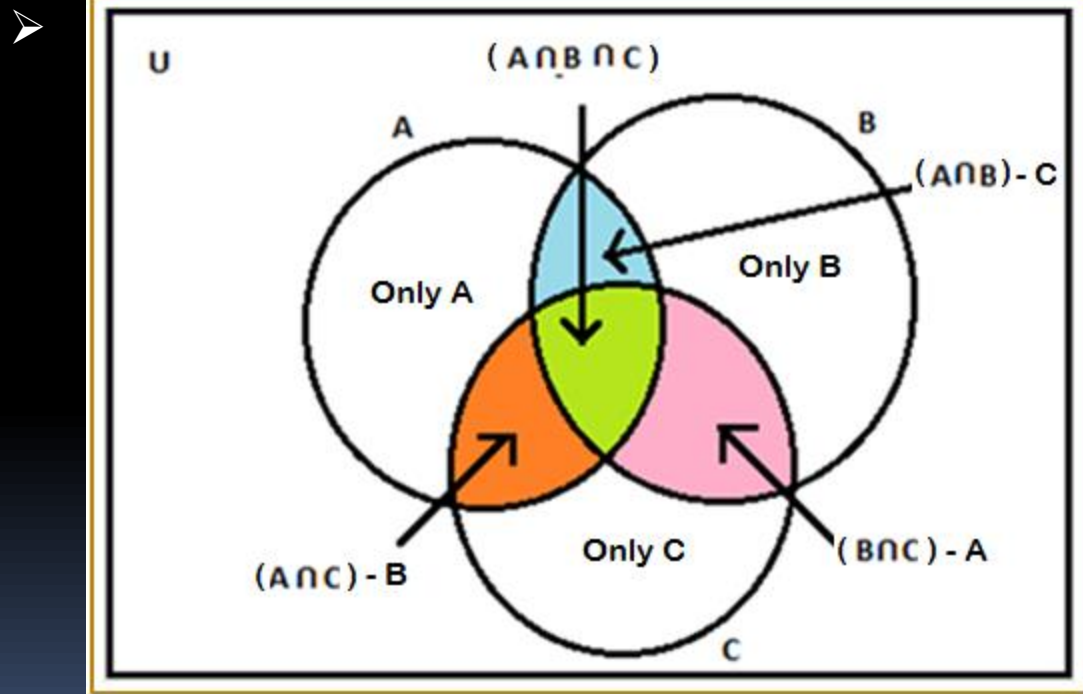
(vii) at least one of three subjects

(iv) physics and chemistry but mathematics (viii) None of three subjects.

QUICK RECAP

➤ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

➤ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$



QUESTION FOR POLL.....

Q. In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Find the number of persons who read neither.

- (a) 110 (b) 90 (c) 550 (d) 290



1. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Solution

Let U be the set of car owners investigated,

M be the set of persons who owned car A

S be the set of persons who owned car B.

Given that $n(U) = 500$, $n(M) = 400$,

$$n(S) = 200 \quad n(S \cap M) = 50.$$

$$\begin{aligned} \text{Then } n(S \cup M) &= n(S) + n(M) - n(S \cap M) \\ &= 200 + 400 - 50 : \\ &= 550 \end{aligned}$$

But $S \cup M \subset U$ implies $n(S \cup M) \leq n(U)$.

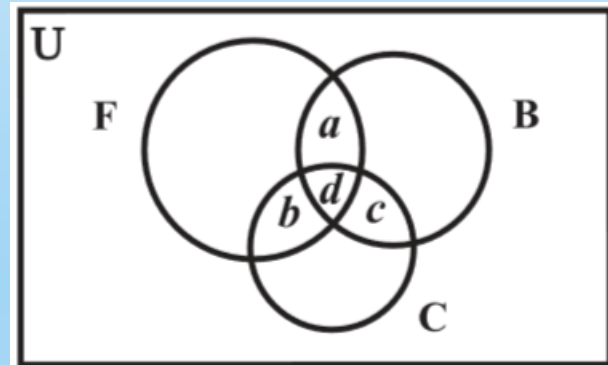
This is a contradiction.

So, the given data is incorrect.

Example 34 A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

Let F , B and C denote the set of men who received medals in football, basketball and cricket, respectively.

$$\begin{aligned} \text{Then } n(F) &= 38, \\ n(B) &= 15, \\ n(C) &= 20 \\ n(F \cup B \cup C) &= 58 \\ n(F \cap B \cap C) &= 3 \Rightarrow d = 3 \end{aligned}$$



$$\begin{aligned} n(F \cup B \cup C) &= n(F) + n(B) + n(C) - n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C) \\ 58 &= 38 + 15 + 20 - [n(F \cap B) + n(F \cap C) + n(B \cap C)] + 3 \\ \Rightarrow n(F \cap B) + n(F \cap C) + n(B \cap C) &= 18 \\ \Rightarrow a + d + b + d + c + d &= 18 \\ \Rightarrow a + b + c &= 9 \end{aligned}$$

$$d = 3$$

Miscellaneous Exercise on Chapter 1

Q.15 In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find: (i) the number of people who read at least one of the newspapers. (ii) the number of people who read exactly one newspaper.

Answer:

Let **H** be the set of people who read newspaper H.

Let **T** be the set of people who read newspaper T.

Let **I** be the set of people who read newspaper I.

Accordingly, $n(H) = 25$, $n(T) = 26$, and $n(I) = 26$

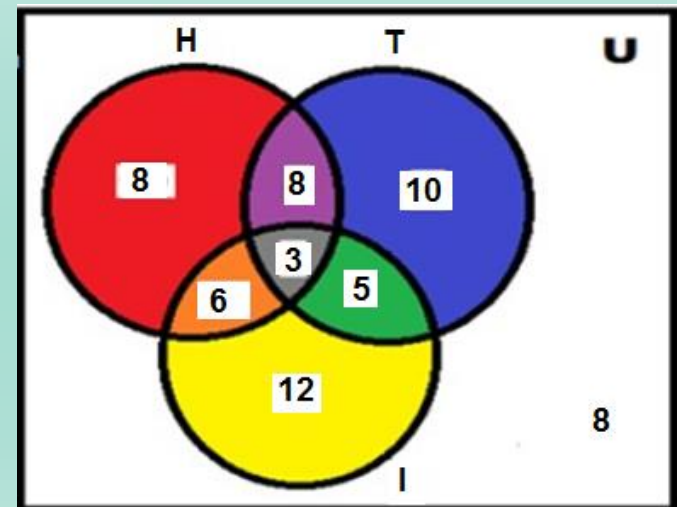
$n(H \cap I) = 9$,

$n(H \cap T) = 11$

$n(T \cap I) = 8$, and

$n(H \cap T \cap I) = 3$

Let **U** be the set of people who took part in the survey.



- (i) The number of people who read at least one of the newspapers = $n(A \cup B \cup C)$
 $= 52$
- (ii) The number of people who read exactly one newspaper = 30

Question 16:

In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Answer

Let A, B, and C be the set of people who like product A, product B, and product C respectively.

$$n(A) = 21,$$

$$n(B) = 26,$$

$$n(C) = 29,$$

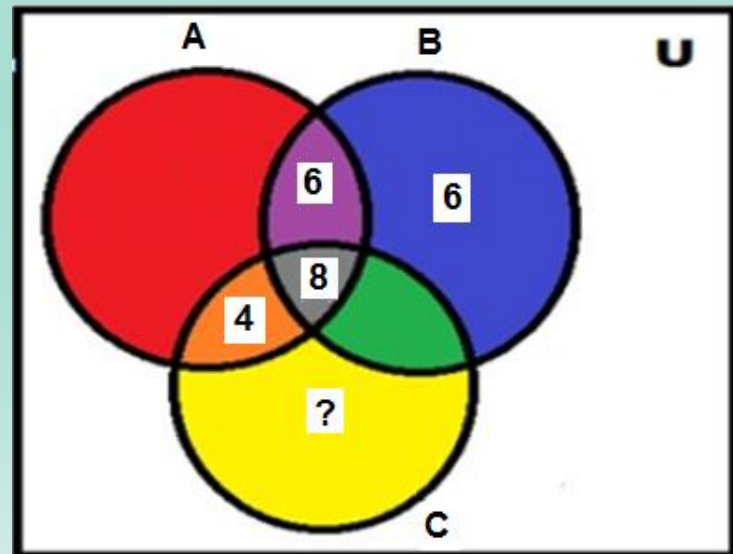
$$n(A \cap B) = 14,$$

$$n(C \cap A) = 12,$$

$$n(B \cap C) = 14,$$

$$n(A \cap B \cap C) = 8$$

Therefore number of people who liked product C only = _____.



SUMMARY

- A **SET** is a well defined collection of objects.
- $B = \{x : x \text{ is a natural number which divides } 21\}$ ----- **Set Builder Form**
 $B = \{1, 3, 7, 21\}$ ----- **Roster Form**
- $\phi \rightarrow$ **Null set / Empty Set**
- $B = \{1, 3, 7, 21\}$ ---- **Finite Set**
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- **Subset**
- **Power Set** ---- Set of all subsets of a set.
- If $n(A) = m$ then $n[P(A)] = 2^m$
- Intervals as subsets of \mathbb{R} : (a, b) , $(a, b]$, $[a, b)$, $[a, b]$
- $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- $A - B = \{x : x \in A \text{ and } x \notin B\}$
- $A^c = \{x : x \in U \text{ and } x \notin A\}$

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- Use of **Venn diagrams** for solving word problems.

ASSIGNMENT QUESTIONS

1. Describe the set in Roster form [1]

$\{x : x \text{ is a two digit number such that the sum of its digit is } 8\}$

2. Are the following pair of sets equal? Give reasons. [1]

$A = \{x : x \text{ is a letter in the word FOLLOW}\}$

$B = \{y : y \text{ is a letter in the word WOLF}\}$

3. If A and B are two given sets, then represent [1]
 $(A - B)'$

Using Venn diagram.

4. $A = \{1, 2, \{3, 4\}, 5\}$ which is incorrect and why. (i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ [1]

5. Fill in the blanks. [1]

(i) $A \cup A' = \dots\dots\dots$ (i) $(A \cup B)' = \dots\dots\dots$

(ii) $(A')' = \dots\dots\dots$ (ii) $(A \cap B)' = \dots\dots\dots$

(iii) $A \cap A' = \dots\dots\dots$

6. List all the element of the set $A = \{x : x \text{ is an integer } x^2 \leq 4\}$ [1]

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8. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ [1]

Find $(A \cap B) \cap (B \cup C)$

9. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form. [1]

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CONTINUED...

14. If $U = \{a, e, i, o, u\}$ $A = \{a, e, i\}$ $B = \{e, o, u\}$ And $C = \{a, i, u\}$ [4]

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Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, Find

(i) the value of x (ii) $n(A \cup B)$

16. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee. [4]

Find (i) how many drink coffee,

(ii) How many drink coffee, but not tea.

17. If A, B, and C, are three sets and U is the universe set such that $n(U) = 1000$, [4]

$n(A) = 300$, $n(B) = 300$ and $n(A \cap B) = 200$ find $n(A' \cap B')$.

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(i) The no. of people who read at least one of the newspapers.

(ii) The no. of people who read exactly one news paper.

19. There are 20 students in a chemistry class and 30 students in a physics class. Find the number of students which are either in physics class or chemistry class in the following cases. [6]

(i) Two classes meet at the same hour

(ii) The two classes met at different hours and ten students are enrolled in both the courses.

20. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. [6]

Find the no. of students that had taken

(i) only chemistry

(v) mathematics and physics but not chemistry

(ii) only mathematics

(vi) only one of the subjects

(iii) only physics

(vii) at least one of three subjects

(iv) physics and chemistry but mathematics (viii) None of three subjects.



Thank You and Happy Learning

