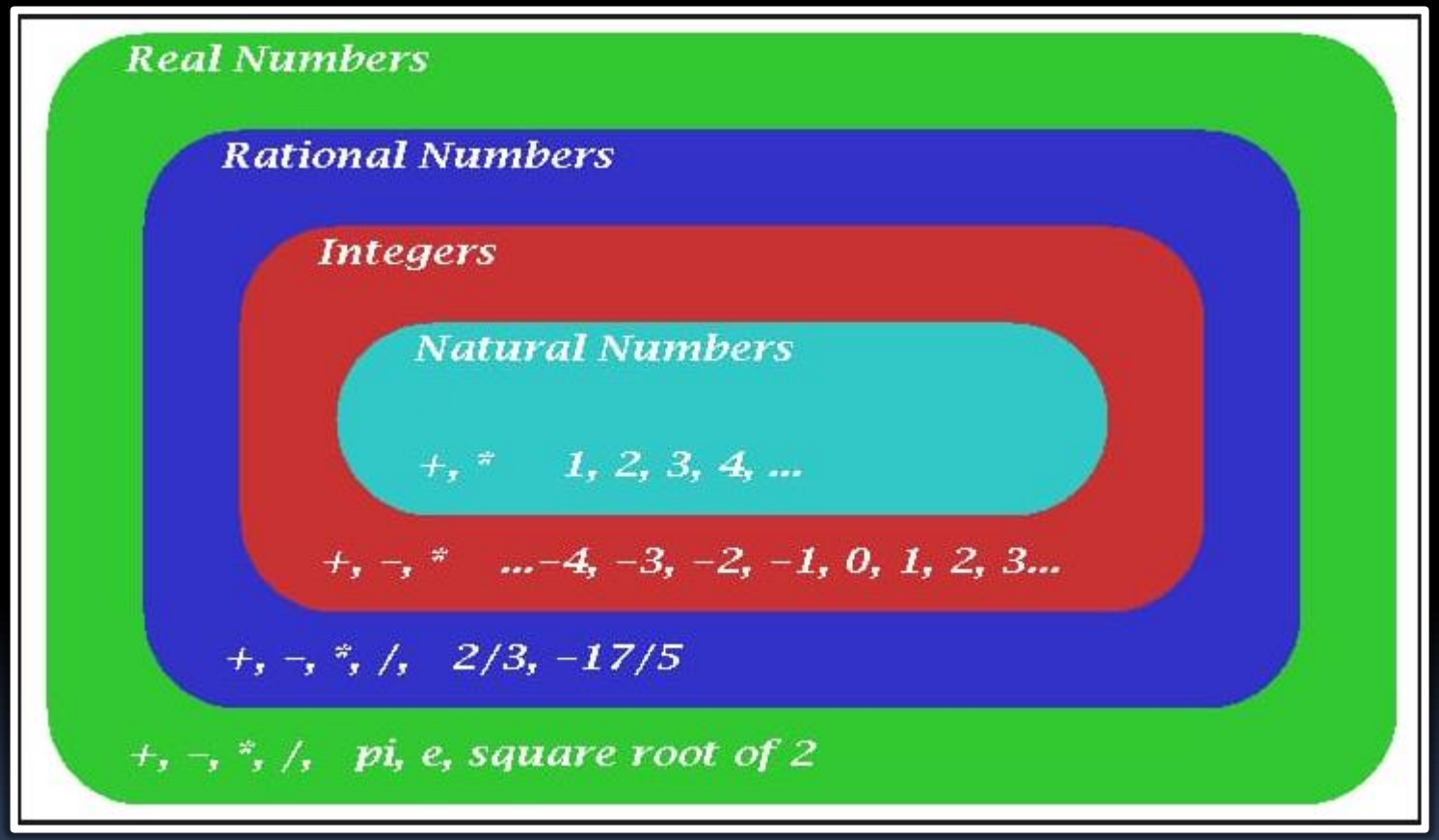
SETS

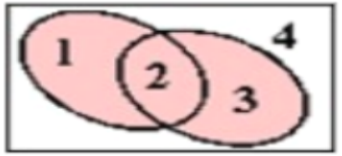
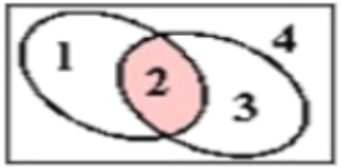
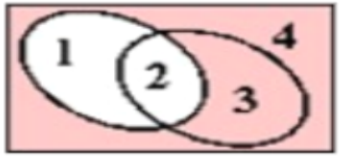


MODULE 3

QUICK RECAP

Let $U = \{1, 2, 3, 4\}$

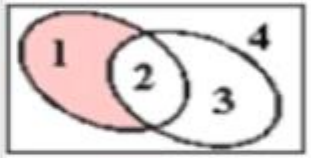
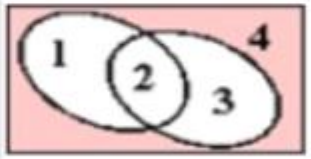
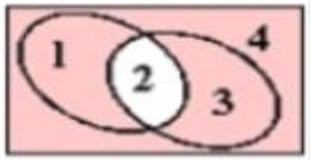
$A = \{1, 2\}$ and $B = \{2, 3\}$

Set Notation	Pronunciation	Meaning	Venn diagram	Answer
$A \cup B$	"A union B"	everything that is in either of the sets		$\{1, 2, 3\}$
$A \cap B$	"A intersect B"	only the things that are in both of the sets		$\{2\}$
A^c or A'	"A complement", or "not A"	everything in the universe outside of A		$\{3, 4\}$

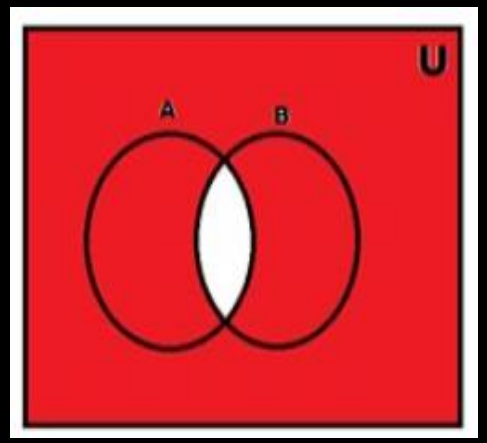
QUICK RECAP

Let U = {1, 2, 3, 4}

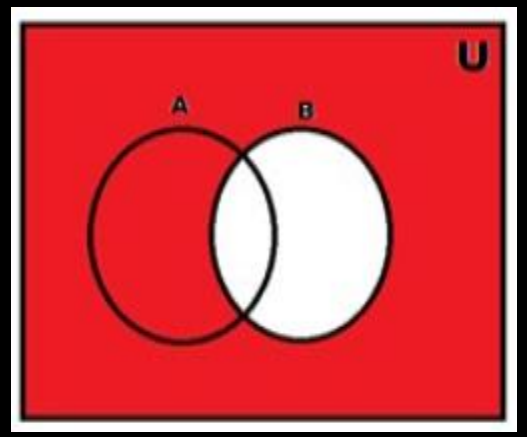
$A = \{1, 2\}$ and $B = \{2, 3\}$

Set Notation	Pronunciation	Meaning	Venn diagram	Answer
$A - B$	"A minus B"	everything in A except for anything in its overlap with B		{1}
$(A \cup B)'$	"not (A union B)"	everything outside A and B		{4}
$(A \cap B)'$	"not (A intersect B)"	everything outside of the overlap of A and B		{1, 3, 4}

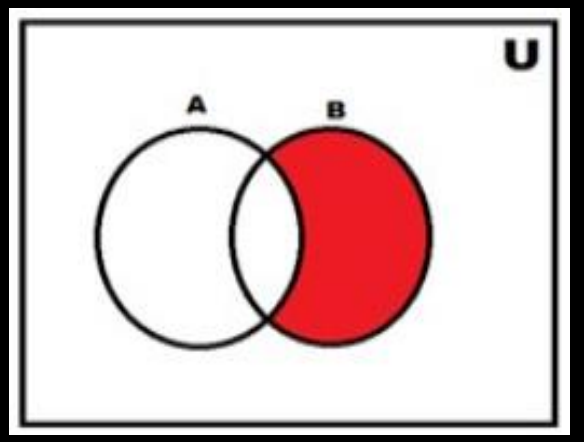
Observe the following Venn diagram and describe by providing the appropriate set operation.



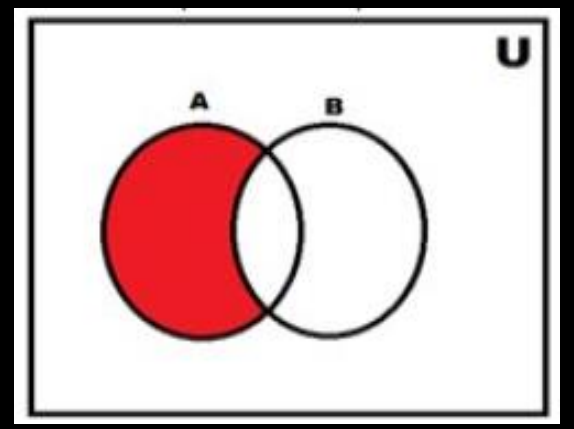
(i)



(ii)



(iii)



(iv)

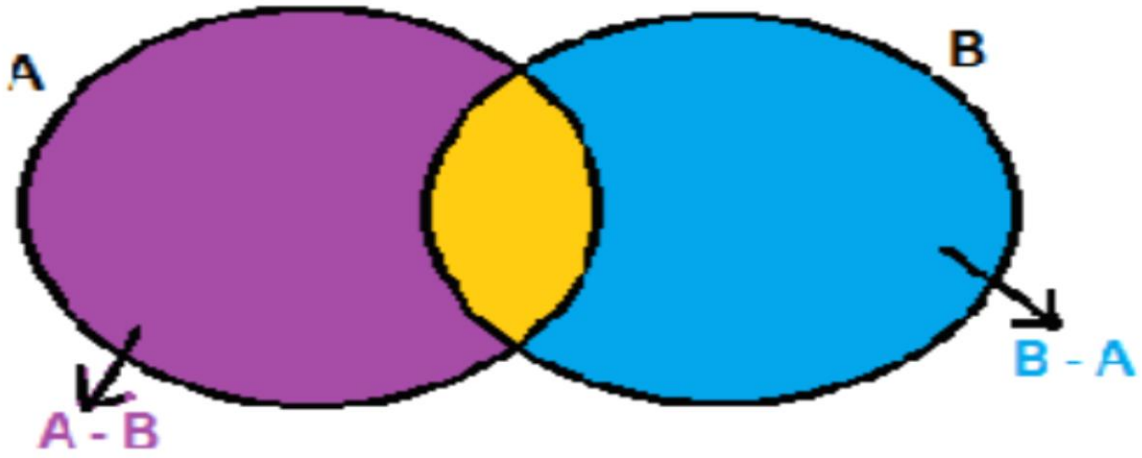
a. B'

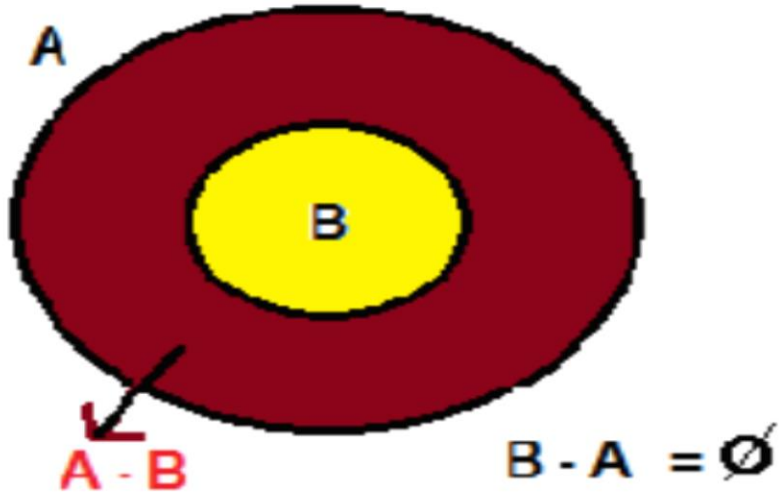
b. $A' \cap B$

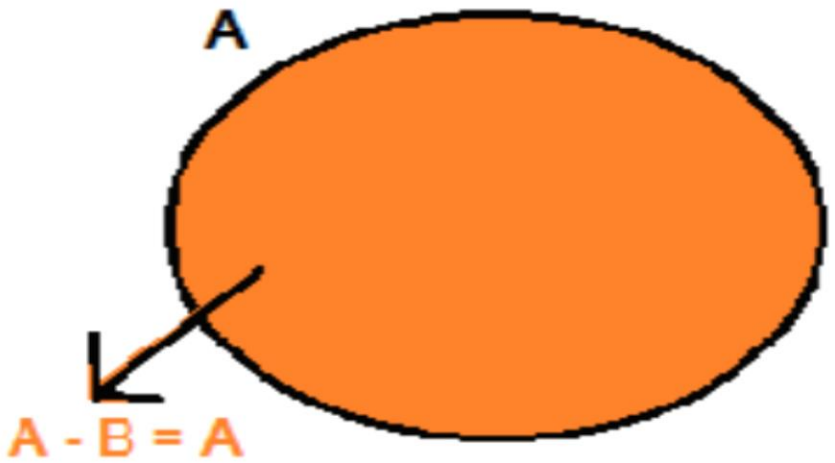
c. $A \cap B'$

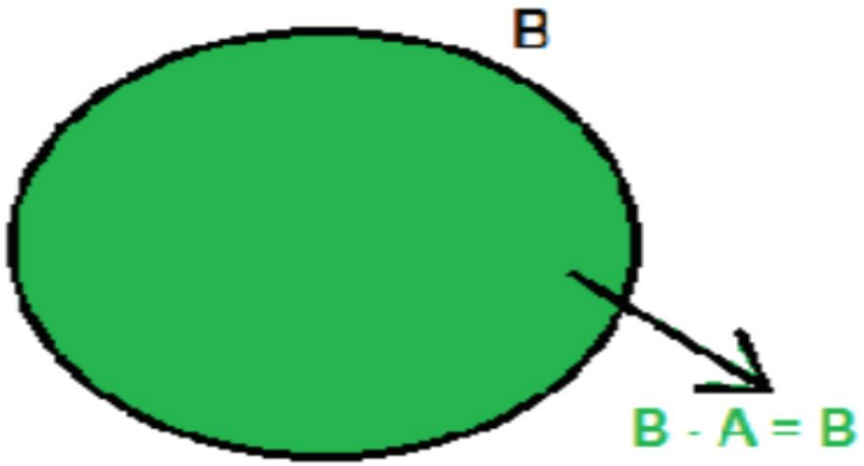
d. $(A \cap B)'$

Possible cases for the difference of two sets:









Properties of Complement

1. Complement laws:

   (i) $A \cup A' = U$ (ii) $A \cap A' = \emptyset$

2. Law of double complementation :

   $(A')' = A$

3. Laws of empty set and universal set

   $\emptyset' = U$ and $U' = \emptyset$

4. **De Morgan's law :** 

   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Proving $(A \cup B)' = A' \cap B'$

De Morgan's Law

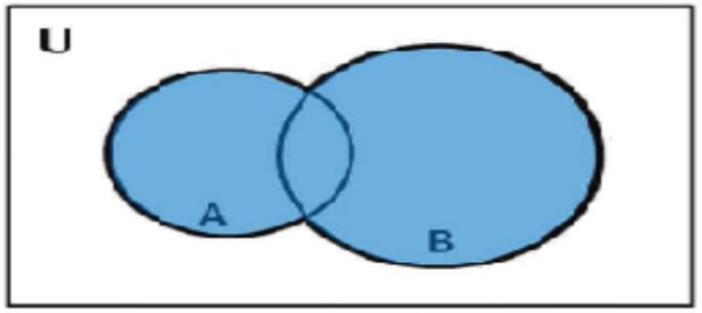
De Morgan's Law are based on complement of sets

1. $(A \cup B)' = A' \cap B'$

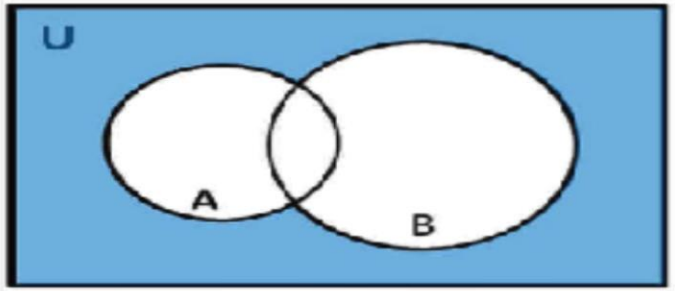
2. $(A \cap B)' = A' \cup B'$

Let us prove the law by Venn Diagrams

$(A \cup B)'$

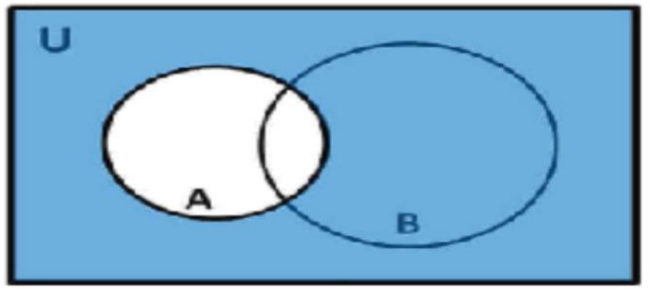


$A \cup B$

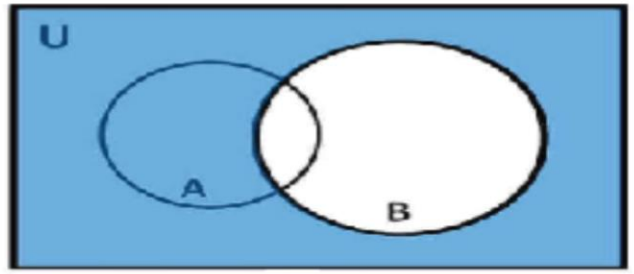


$(A \cup B)'$

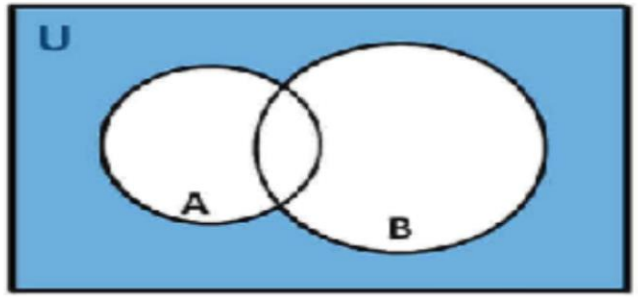
$A' \cap B'$



A'



B'



$A' \cap B'$

$\therefore (A \cup B)' = A' \cap B'$

SOME BASIC RESULTS ABOUT CARDINAL NUMBER

In today's module, we will go through some practical problems related to our daily life. The formulae derived in this section will also be used in subsequent Chapter on Probability

Let A and B be finite sets.
If $A \cap B = \phi$, then
$n(A \cup B) = n(A) + n(B)$

NOTE:
$n(A \cup B) \leq n(U)$

In general, if A and B are finite sets, then

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ... (1)

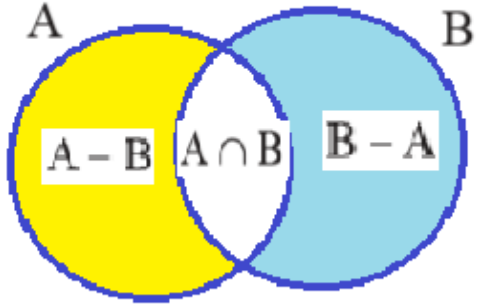
Note that the sets $A - B$, $A \cap B$ and $B - A$ are disjoint and their union is $A \cup B$

Therefore

$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$= n(A - B) + n(A \cap B) + n(B - A) + n(A \cap B) - n(A \cap B)$

$= n(A) + n(B) - n(A \cap B)$, which verifies (1)



If $A \cap B = \phi$, then (i) $n(A \cup B) = n(A) + n(B)$ ... (1)

If A and B are finite sets and $A \cap B \neq \phi$, then

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Word problems on sets are solved here to get the basic ideas how to use the properties of union and intersection of sets.

1) **Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A ∪ B) = 36, find n(A ∩ B).**

Solution:

Using the formula n(A ∪ B) = n(A) + n(B) - n(A ∩ B).

then n(A ∩ B) = n(A) + n(B) - n(A ∪ B)

= 20 + 28 - 36

= 48 - 36

= 12

2). There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

- **When two classes meet at different hours and 12 students are enrolled in both activities.**
- **When two classes meet at the same hour.**

Solution: $n(A) = 35, \quad n(B) = 57, \quad n(A \cap B) = 12$

(Let **A** be the set of students in art class and **B** be the set of students in dance class.)

(i) When 2 classes meet at different hours $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 35 + 57 - 12$

$= 92 - 12$

$= 80$

(ii) When two classes meet at the same hour, $A \cap B = \emptyset$

So, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A) + n(B)$

$= 35 + 57$

$= 92$

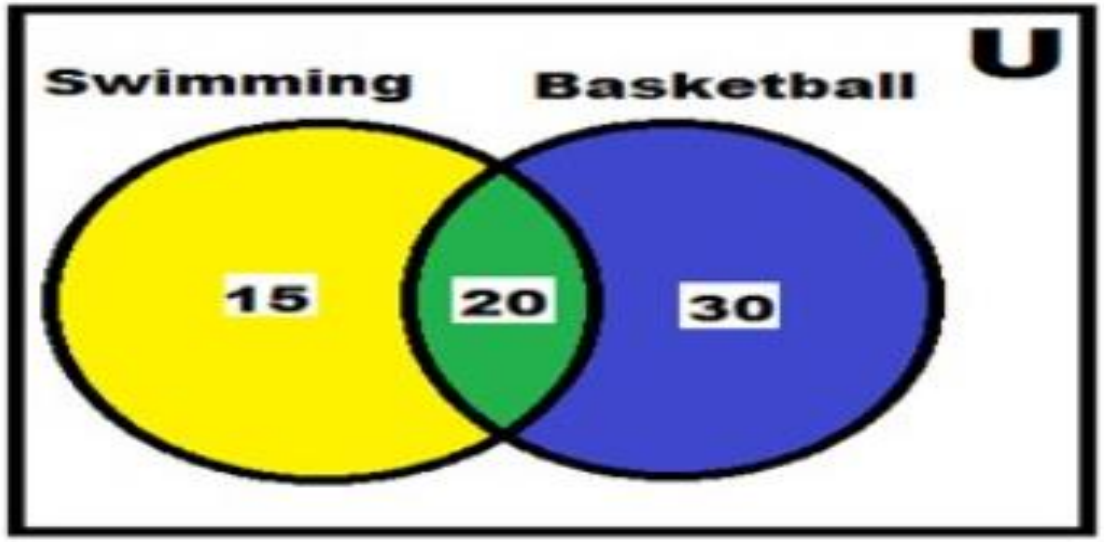
3. If X and Y are two sets such that X ∪ Y has 50 elements, X has 28 elements and Y has 32 elements, how many elements does X ∩ Y have ?

CAN YOU TRY THIS ???

4.

The Venn diagram is also used to analyse different relationship that exists between two or more objects.

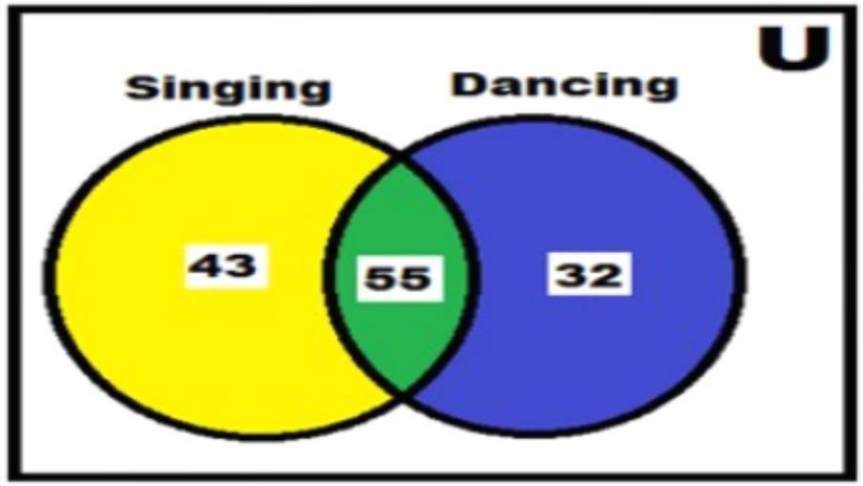
The Venn diagram below shows student involvement in two sports. If 100 students were surveyed, how many students were:



1. Into swimming but not basketball?
2. Into both swimming and basketball?
3. Into either swimming or basketball?
4. Neither into swimming nor basketball?

TRY THIS:

5. 140 students were surveyed. The Venn diagram shows the number of students who enjoy singing and/or dancing. How many students enjoy:



1. Dancing but not singing?
2. Singing or dancing?
3. Singing and dancing?
4. Neither singing nor dancing?

6.

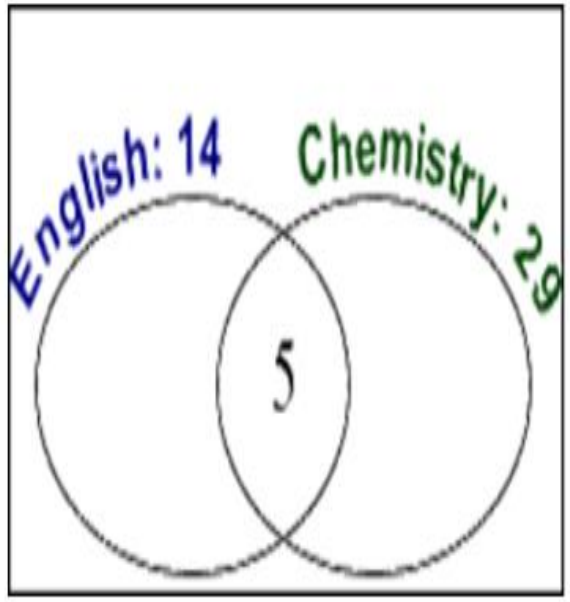
Venn Diagram in Word Problems

Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes:

- How many students are taking English Composition only?
- How many students are taking Chemistry only?
- How many students are in neither class?
- How many are in either class?

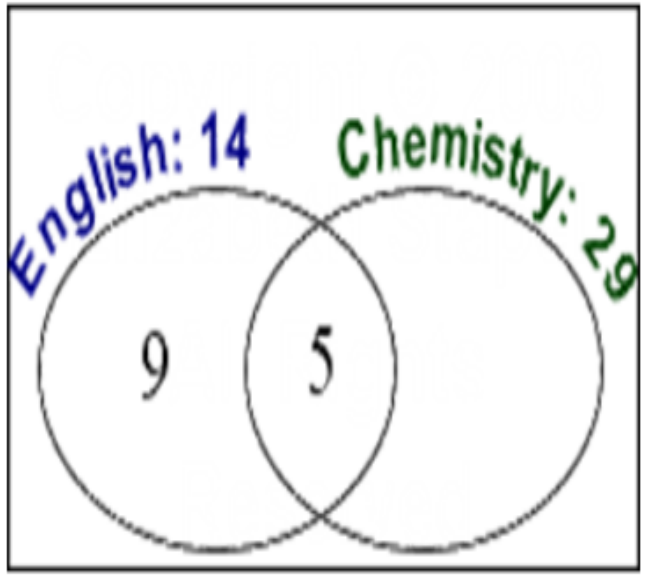
STEP 1:

Since five students are taking both classes, put "5" in the overlap:



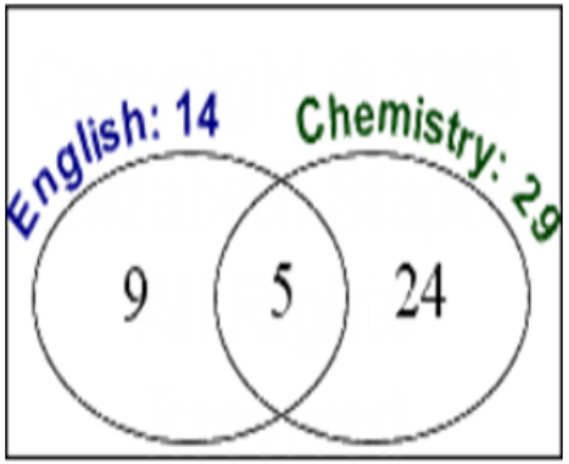
STEP 2:

We have now accounted for five of the 14 English students, leaving nine students taking English but not Chemistry, so we put "9" in the "English only" part of the "English" circle:



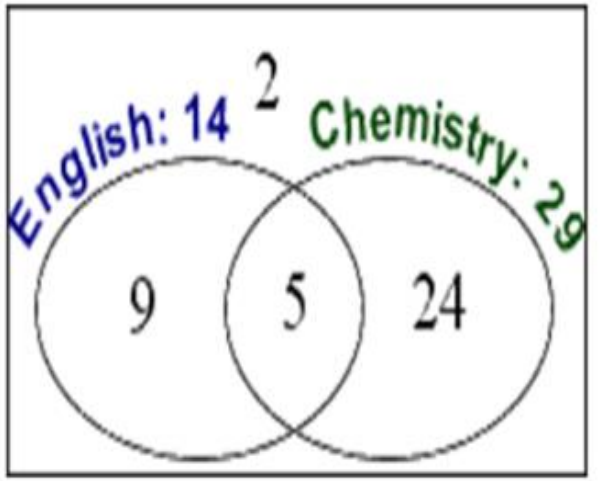
STEP 3:

We have also accounted for five of the 29 Chemistry students, leaving 24 students taking Chemistry but not English, so we put "24" in the "Chemistry only" part of the "Chemistry" circle:



STEP 4:

This tells us that a total of $9 + 5 + 24 = 38$ students are in either English or Chemistry (or both). This leaves two students unaccounted for, so they must be the ones taking **neither** class.



- **No. of students taking English Composition only - 9**
- **No. of students taking Chemistry only - 24**
- **No. of students in neither class - 2**
- **No. of students in either class - 38**

7.

Consider the following example:

Question: In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like either of them and how many like neither?

Solution:

Total number of stude $n(U) = 100$ ɔo

Number of science students, $n(S) = 35$

Number of math students, $n(M) = 45$

Number of students who like both, $n(M \cap S) = 10$

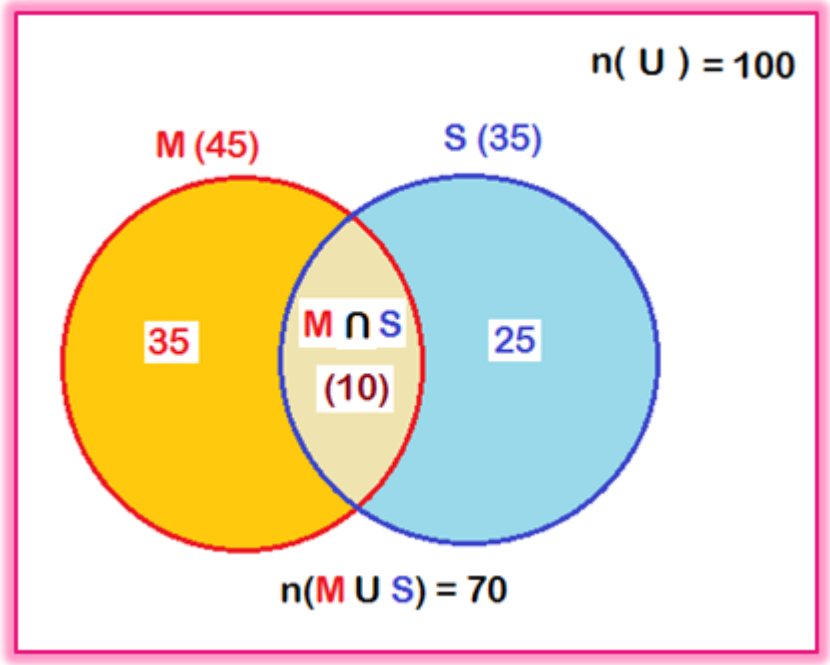
Number of students who like either of them,

$n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$\rightarrow 45+35-10 = 70$

Number of students who like neithe $n(U)$ (μ) $- n(M \cup S) = 100 - 70 = 30$

The easiest way to solve problems on sets is by drawing Venn diagrams, as shown below.



POLLING TIME !!!

8. If $n(X - Y) = 18$, $n(X \cup Y) = 70$ and $n(X \cap Y) = 25$, then find $n(Y)$.
A) 27 B) 43 C) 52 D) 45

H.W.

1. In a class of 60 students, 40 students like math, 36 like science, 24 like both the subjects. Find the number of students who like
(i) Math only, (ii) Science only (iii) Either Math or Science
(iv) Neither Math nor science.

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

3. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

ASSIGNMENT QUESTIONS

1. Describe the set in Roster form [1]

   $\{x : x$ is a two digit number such that the sum of its digit is 8 $\}$

2. Are the following pair of sets equal? Give reasons. [1]

   A = { x:x is a letter in the word FOLLOW}

   B = { y:y is a letter in the word WOLF}

3. If A and B are two given sets, then represent $(A - B)'$ [1]
   Using Venn diagram.

4. A = {1, 2, {3, 4}, 5} which is incorrect and why. (i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ [1]

5. Fill in the blanks. [1]

   (i) $A \cup A' =$ -------- (i) $(A \cup B)' =$ ---------

   (ii) $(A')' =$ --------- (ii) $(A \cap B)' =$ ---------

   (iii) $A \cap A' =$ --------

6. List all the element of the set A = { x : x is an integer $x^2 \leq 4$} [1]

7. From the sets given below pair the equivalent sets. [1]

   A = { 1, 2, 3}, B = {x, y, z, t}, C = {a, b, c} D = {0, a}

8. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} [1]

   Find $(A \cap B) \cap (B \cup C)$

9. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form. [1]

10. Write down all the subsets of the set {1,2,3} [1]

11. Write down all possible proper subsets of the set {1, {2}}. [1]

12. State whether each of the following statement is true or false. [1]

    (i) {2, 3, 4, 5} and {3, 6} are disjoint

    (ii) {2, 6, 10} and {3, 7, 11} are disjoint sets

13. Write the following as interval [1]

    (i) $\{x : x \in R, -4 < x \leq 6\}$

    (ii) $\{x : x \in R, 3 \leq x \leq 4\}$

CONTINUED...

14. If $U = \{a, e, i. o. u\}$ $A = \{a, e, i\}$ $B = \{e, o, u\}$ And $C = \{a, i, u\}$ [4]
Then verity that $A \cap (B - C) = (A \cap B) - (A \cap C)$

15. A and B are two sets such that $n(A - B) = 20 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x + 1$. [4]
Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, Find
(i) the value of x (ii) $n(A \cup B)$

16. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not [4]
coffee, each member drinks tea or coffee.
Find (i) how many drink coffee,
(ii) How many drink coffee, but not tea.

17. If A, B, and C, are three sets and U is the universe set such that n(U) = 1000, [4]
$n(A) = 300$, $n(B)$ 300 and $n(A \cap B) = 200$ find $n(A' \cap B')$.

18. In a survey of 60 people, it was found that 25 people read news paper H, 26 [6]
read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H
and T, 8 read both T and I, 3 read all three newspaper. Find
(i) The no. of people who read at least one of the newspapers.
(ii) The no. of people who read exactly one news paper.

19. These are 20 students in a chemistry class and 30 students in a physics class. Find [6]
the number of students which are either in physics class or chemistry class in the following cases.
(i) Two classes meet at the same hour
(ii) The two classes met at different hours and ten students are enrolled in both the courses.

20. In a survey of 25 students, it was found that 15 had taken mathematics, 12had taken [6]
physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects.
Find the no. of students that had taken
(i) only chemistry
(ii) only mathematics
(iii) only physics
(iv) physics and chemistry but mathematics
(v) mathematics and physics but not chemistry
(vi) only one of the subjects
(vii) at least one of three subjects
(viii) None of three subjects.

Thank You and Happy Learning