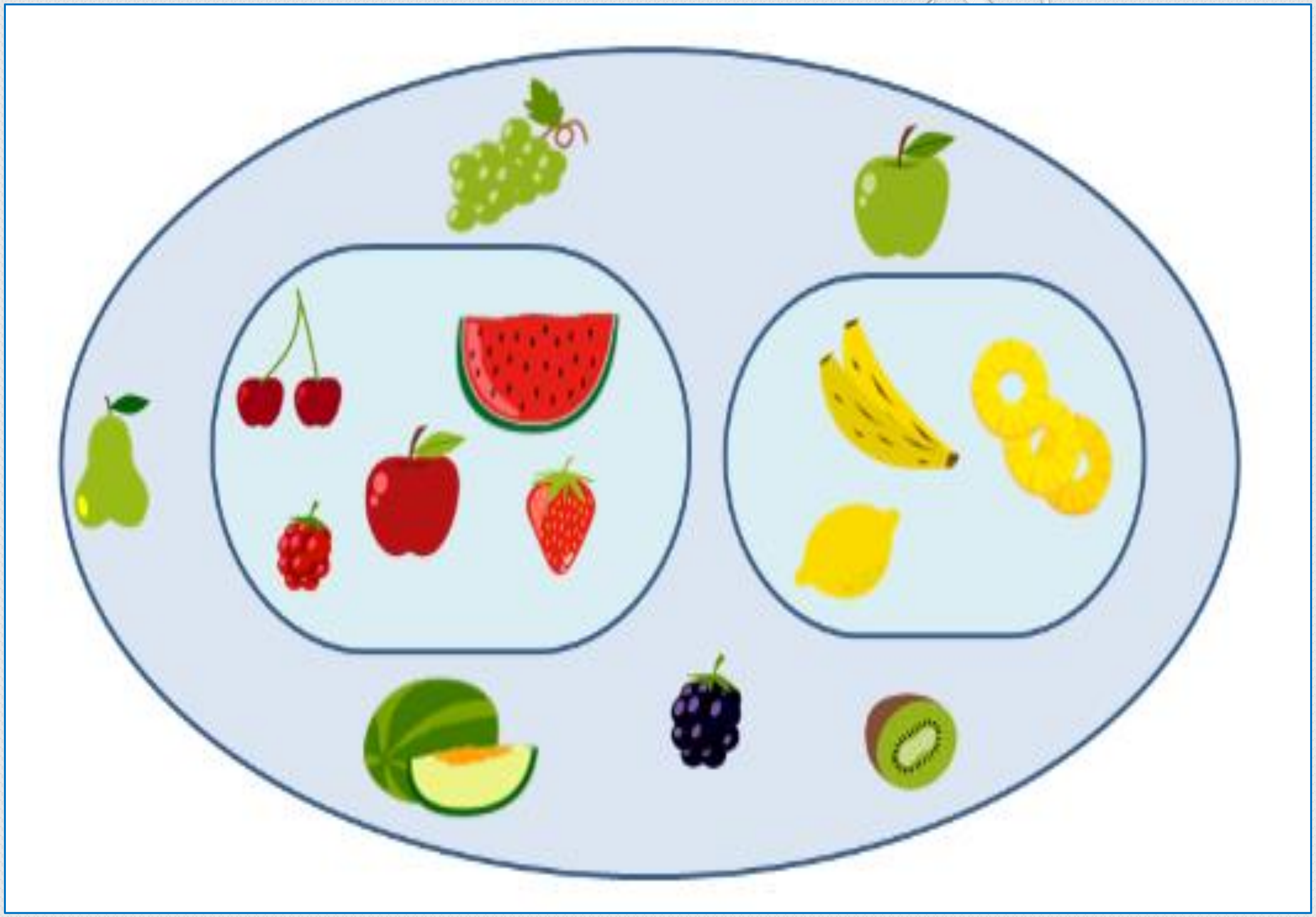
CLASS:XI

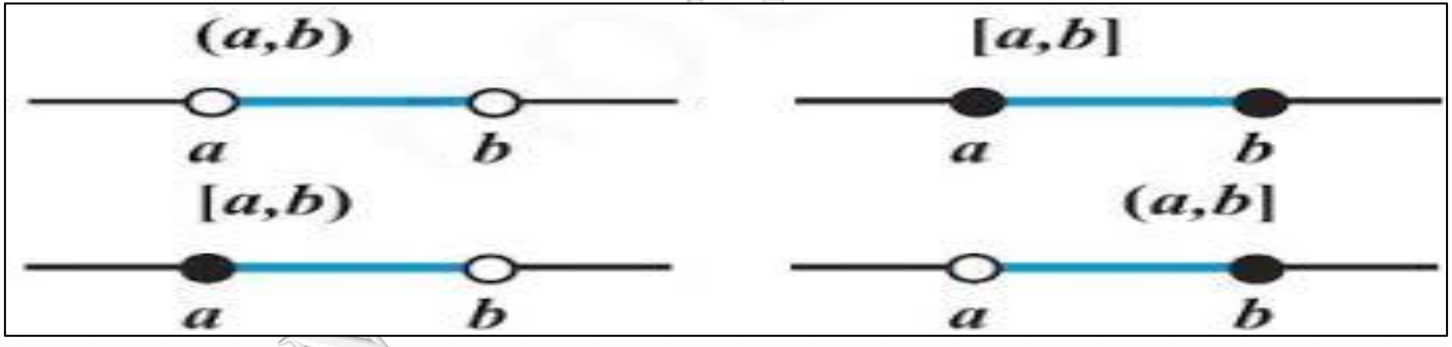
SETS

MODULE - 2



RECAP :

- A set A is said to be a subset of a set B if every element of A is also an element of B.
- Every set is a subset of itself.
- Empty set is a subset of every set.
- On real number line, various types of intervals can be described as subsets of **R**.



- If the number of elements in a set is m, then the number of subsets is 2^m

POWER SET:

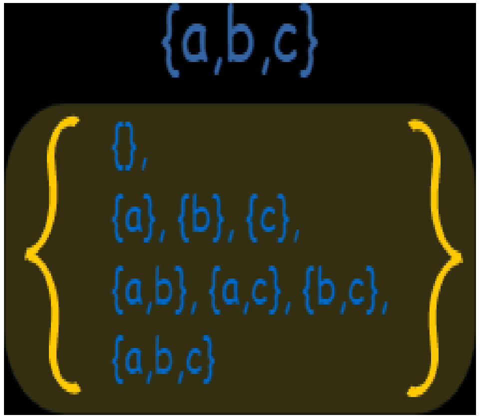
All The Subsets

- For the set {a,b,c}:
- The empty set {} is a subset of {a,b,c}

  And these are subsets: {a}, {b} and {c}

  And these are also subsets: {a,b}, {a,c} and {b,c}
- And {a,b,c} is a subset of {a,b,c}

  And altogether we get the **Power Set** of {a,b,c}:
- P(S) = { {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} }



Let’s go back to the ice cream example …..

We have four flavors of ice cream: **banana, chocolate, lemon, and strawberry**. How many different ways can we have them?

Let's use letters for the flavors: {b, c, l, s}. Hence, the result is…..

P = { {}, {b}, {c}, {l}, {s}, {b,c}, {b,l}, {b,s}, {c,l}, {c,s}, {l,s}, {b,c,l}, {b,c,s}, {b,l,s}, {c,l,s}, {b,c,l,s} }

LET'S UNDERSTAND:

TEN BEST FRIENDS….

Imagine you have a set made up of your ten best friends:

$\{Raj, Kiran, Vishal, Niraj, Suresh, Gopi, Kartik, Yash, Arjun, Manoj\}$

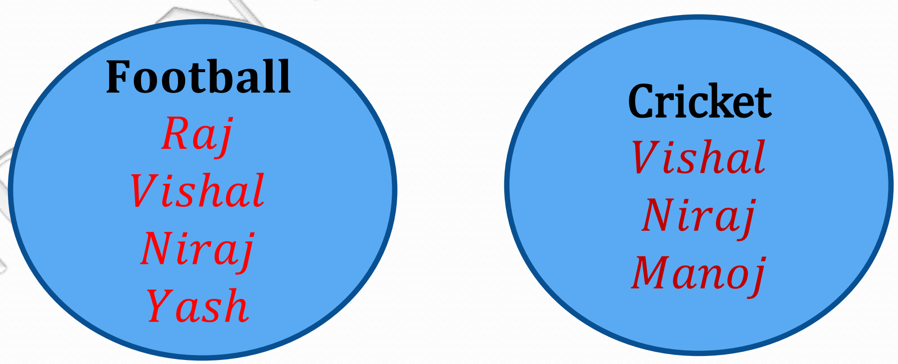
Now let's say that Raj, $Vishal, Niraj$ and $Yash$ play **Football**.

∴Football= $\{Raj, Vishal, Niraj, Yash\}$

And $Vishal, Niraj$ and $Manoj$ play **Cricket**.

∴Cricket= $\{Vishal, Niraj, Manoj\}$

We can put their names in two separate circles, representing two sets:



operations on sets (UNION):

- You can now list your friends who play **Football or Cricket**.

Not everyone is in this set…..only your friends who play Football or Cricket (or both). In other words, we combine the elements of the two sets.

This is called UNION of two sets, denoted by the symbol ∪.

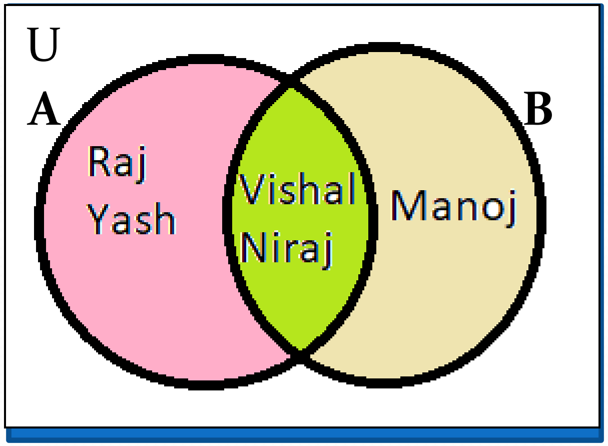
$$\therefore \textbf{Football} \cup \textbf{Cricket} = \{Raj, Vishal, Niraj, Yash, Manoj\}$$

We can show that in a "Venn Diagram"

In a Venn Diagram:

➡ The Universal set (U) is represented by a rectangle, and its subsets by circles.

➡ Let A and B be two sets, then the union of A and B, written as A∪B, is the set of all elements which are members of the set A or B or both.



Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

operations on sets (INTERSECTION):

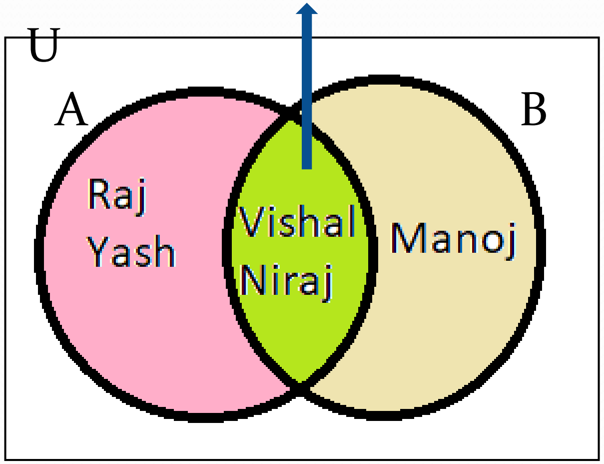
Intersectionis when you must be in BOTH sets.

Inour example, that means, those who play both Football and Cricket.

This is called INTERSECTION of two sets, denoted by the symbol ∩.

$\therefore$ **Football** ∩ **Cricket** $= \{Vishal, Niraj\} A \cap B$

Let A and B be two sets, then the intersection of A and B, written as A ∩ B, is the set of all those elements which are common to both A and B.

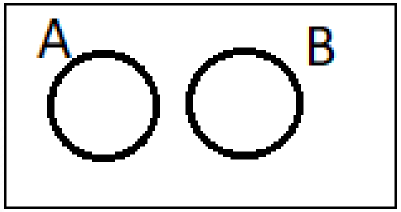


Thus, $A \cap B = \{x : x \in A \;\; and \;\; x \in B\}$

NOTE: If A and B are two sets such that $A \cap B = \emptyset$, then A and B are called disjoint sets.

For eg: A = set of odd natural numbers

B = set of even natural numbers



PROPERTIES OF UNION AND INTERSECTION OF SETS:

- **UNION OF SETS**

(i) $A \cup B = B \cup A$(Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \emptyset = A$(Existence of identity)

(iv) $A \cup A = A$ (Idempotent law)

(v) $A \cup U = U$ and $\emptyset \cup U = U$

- **INTERSECTION OF SETS**

(i) $A \cap B = B \cap A$(Commutative law)

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(iii) $A \cap \emptyset = \emptyset$(Existence of identity)

(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap U = A$ and $\emptyset \cap U = \emptyset$

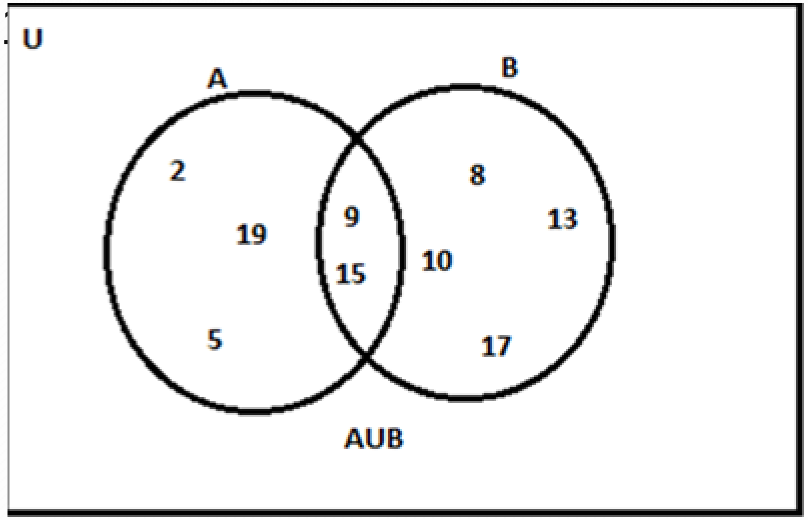
LET'S PRACTICE……

Example 1: Let U be a universal set consisting of all the natural numbers less than 20 and set A and B be a subset of U defined as A={2,5,9,15,19} and B={8,9,10,13,15 ,17}. Find A∪B and A∩B.

Solution: Given
U={1,2,3,4,5,6,7,8,9,10,11,12,13,14,

- A={2,5,9,15,19}
- B={8,9,10,13,15,17}
- A∪B={2,5,8,9,10,13,15,17,19}

  A∩B = {9,15}



Let's practice....

Example 2:

Given $A = \{x : x$ is an integer and $1 \leq x \leq 5\} \rightarrow \text{A} = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 17\}$ and

$C = \{1, 2, 3 \ldots\}$. Find:

(i) $A \cap B = \{3, 4, 5\}$

(ii) $A \cap C = \{1, 2, 3, 4, 5\}$

(iii) $A \cup B = \{1, 2, 3, 4, 5, 17\}$

(iv) $A \cup C = \{1, 2, 3 \ldots\} = C$

(v) $\text{B} \cap C = \{3, 4, 5, 17\} = B$

(vi) $\text{B} \cup C = \{1, 2, 3 \ldots\} = C$

Example 3:

If $A = \{x : x$ is a natural number$\}$, $B = \{x : x$ is an even natural number$\}$, $C = \{x : x$ is an odd natural number$\}$

Find:

(i) $A \cap B = \{1,2,3,4, \ldots\} \cap \{2,4,6,8, \ldots\}$
$= \{2,4,6,8, \ldots\} = B$

(ii) $B \cup C = \{2,4,6 \ldots\} \cup \{1,3,5, \ldots\}$
$= \{1,2,3, \ldots\} = A$

(iii) $A \cap C =$ ______

(iv) $B \cap C =$ ______

(v) $A \cup B =$ ______

HOMEWORK

1) Write the power set of the following sets: (i)$A = \{1,2,3\}$ (ii)$B = \{\{a, b\}, c\}$.

2) Rewrite the set $A = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots\right\}$ set-builder form.

3) If $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$, find:

(i) $A \cup B \cup C$ (ii)$A \cup B \cup D$ (iii)$\mathrm{B} \cup C \cup D$

4) If $A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$ and $D = \{15, 17\}$, find: (i) $A \cap C \cap D$ (ii) $A \cap (B \cup D)$ (iii) $(A \cap B) \cap (B \cup C)$

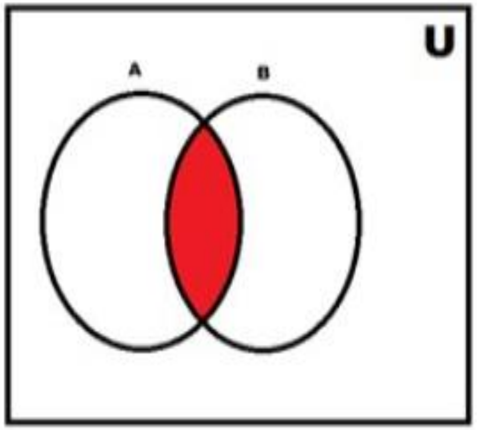
5) Which of the following pairs of sets are disjoint?

(i) $\{x: x$ is a natural number and $4 \le x \le 6\}$ and $\{1, 2, 3, 4\}$

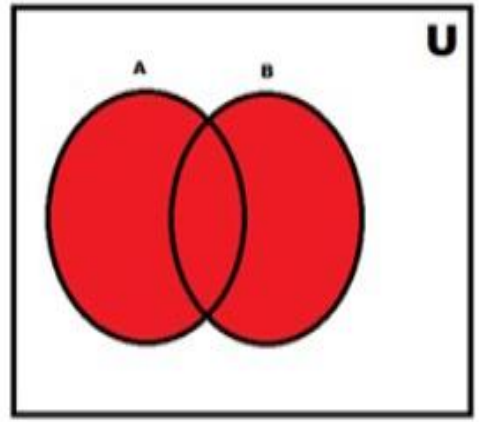
(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x: x \text{ is an even integer}\}$ and $\{x: x \text{ is an odd integer}\}$

RECAP



$A \cap B$



$A \cup B$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

If $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$ then find:

(i) $(B \cup C)$

(ii) $(A \cap B)$

Rule

Distributive Laws

For any three sets A, B, & C, the following statements are true:

- $\cap$ is distributive over $\cup$ from the left & the right.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

- $\cup$ is distributive over $\cap$ from the left & the right.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$B \cup C = \{3, 4, 5, 6\} \cup \{6, 7, 8\} = \{3, 4, 5, 6, 7, 8\}$

$A \cap (B \cup C) = \{1, 2, \mathbf{3}, \mathbf{4}\} \cap \{\mathbf{3}, \mathbf{4}, 5, 6, 7, 8\} = \{3, 4\}$

$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$

$A \cap C = \{1, 2, 3, 4\} \cap \{6, 7, 8\} = \{\} = \emptyset$

$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \emptyset = \{3, 4\}$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$B \cap C = \{3, 4, 5, \mathbf{6}\} \cap \{\mathbf{6}, 7, 8\} = \{6\}$

$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{6\} = \{1, 2, 3, 4, 6\}$

$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4\} \cup \{6, 7, 8\} = \{1, 2, 3, 4, 6, 7, 8\}$

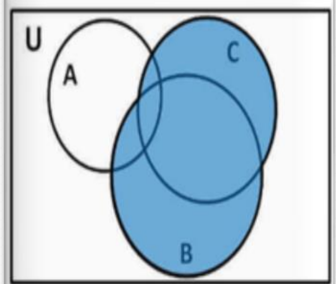
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 6, 7, 8\} = \{1, 2, 3, 4, 6\}$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

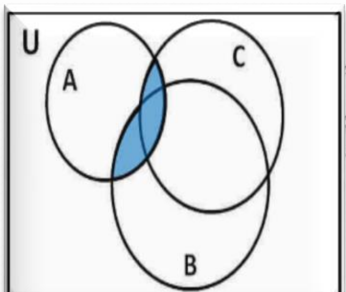
Distributive law of sets

We have to prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using Venn Diagram.

$A \cap (B \cup C)$

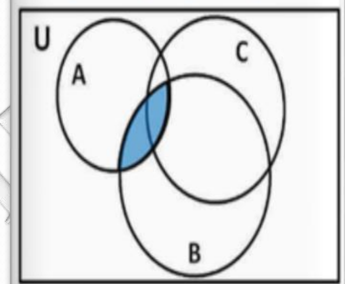


$(B \cup C)$

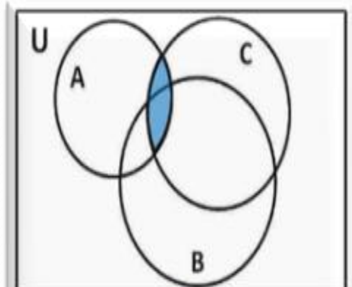


$A \cap (B \cup C)$

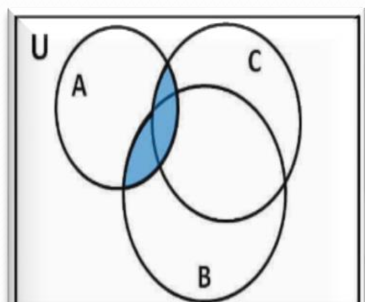
$(A \cap B) \cup (A \cap C)$



$(A \cap B)$



$(A \cap C)$



$(A \cap B) \cup (A \cap C)$

L.H.S. = R.H.S. Hence Proved

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Ten Best Friends

You could have a set made up of your ten best friends:

- {alex, blair, casey, drew, erin, francis, glen, hunter, ira, jade}

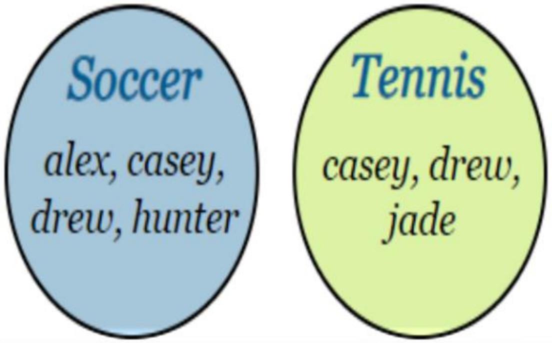
Each friend is an "element" (or "member") of the set.

Now let's say that alex, casey, drew and hunter play **Soccer**.

Soccer = {alex, casey, drew, hunter}

And casey, drew and jade play **Tennis**.

Tennis = {casey, drew, jade}



Difference

You can also "subtract" one set from another.

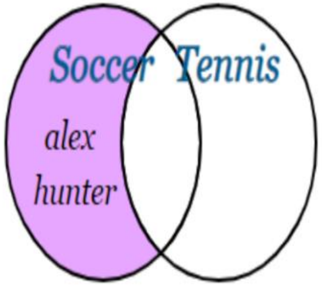
For example, taking Soccer and subtracting Tennis

means people that **play Soccer but NOT Tennis** ...

which is alex and hunter.

And this is how we write it: Soccer − Tennis = {alex, hunter}

In a Venn Diagram:

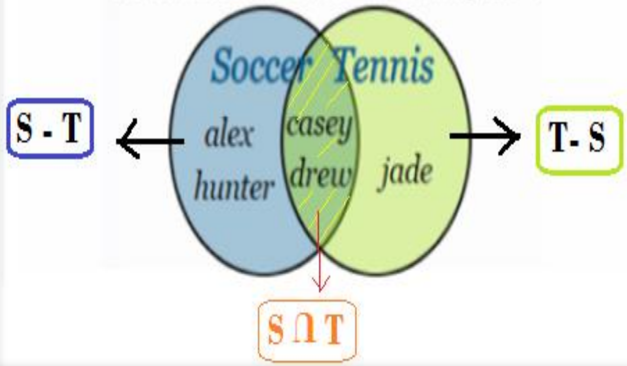


Similarly Tennis − Soccer = { jade }

Soccer ∩ Tennis = {casey, drew}

Soccer ∪ Tennis = {alex, casey, drew, hunter, jade}

We can show that in a "Venn Diagram":



Difference of sets

Difference of set A and B means all elements of set A which are not in B

Let A = {1, 2, **3**, **4** }, B = { **3**, **4** , 5, 6}

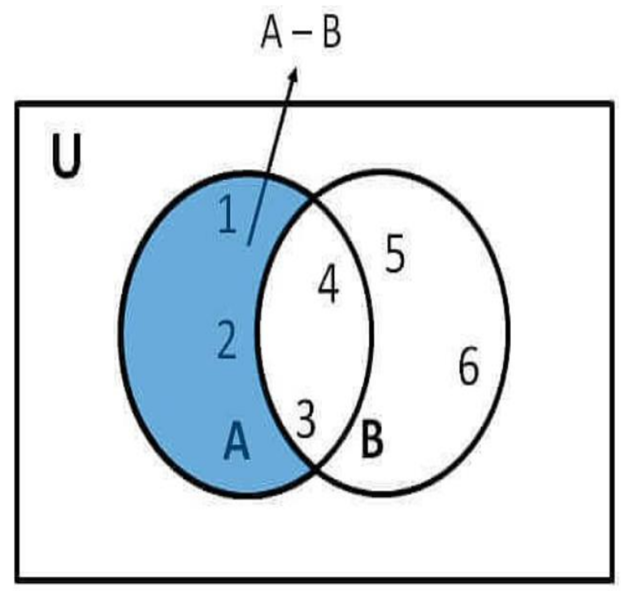
A - B = {1, 2}

For B – A

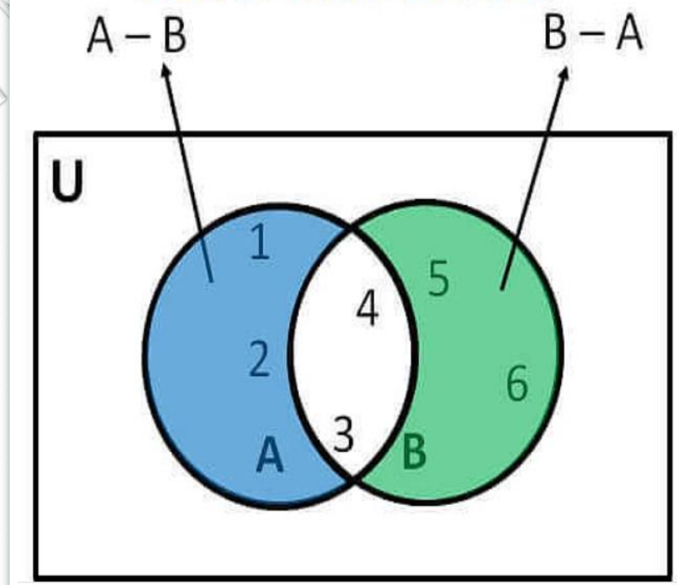
B – A = { **3, 4** , 5, 6} – {1, 2, **3, 4** }

B – A = {5, 6}

As B – A = All elements of B which are not in A



Difference of Sets



Symbolically,

A - B = { x : x ∈ A and x ∉ B } and vice versa.

Example

Let A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find $A \cap B$, $A - B$ and $B - A$.

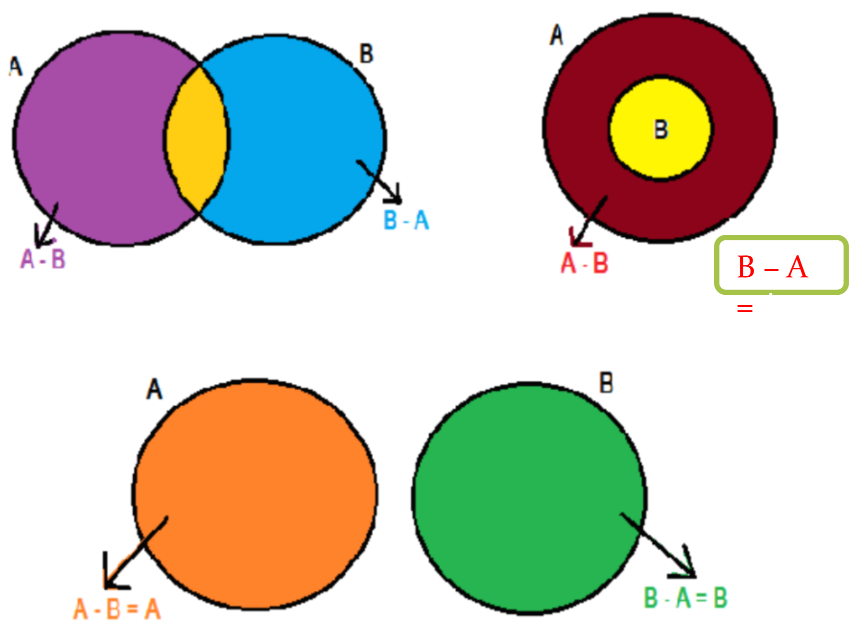
$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 6\}$

$A - B =$ **{1, 3, 5}**

$B - A =$ **{8}**

Possible cases for the difference of two sets:



The set of elements of U which are not elements of set A is called the complement of A.

A^c or A'

'the complement of set A'

Let A = {1, 2, 3, 4} , B = {3, 4, 5, 6}, C = {6, 7, 8}

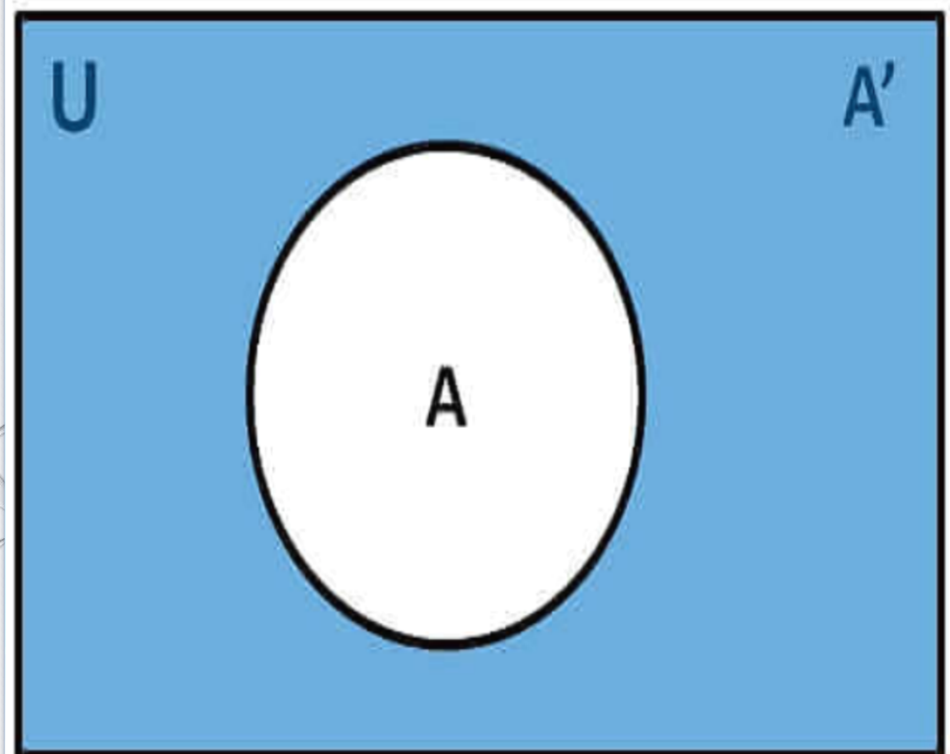
and Universal set = U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$A' = U - A$

$A' = \{ \mathbf{1, 2, 3, 4}, 5, 6, 7, 8, 9, 10\} - \{ \mathbf{1, 2, 3, 4} \}$

$A' = \{5, 6, 7, 8, 9, 10\}$

Complement of Set



$$A' = \{x \mid x \in U \text{ and } x \notin A\}$$

Properties of Complement

1. Complement laws:

   (i) $A \cup A' = U$ (ii) $A \cap A' = \emptyset$

2. Law of double complementation :

   $(A')' = A$

3. Laws of empty set and universal set

   $\emptyset' = U$ and $U' = \emptyset$

4. **De Morgan's law** :

   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Example:

1. $U = \{3, 4, 5, 7, 8, 9, a, b\}$

   $A = \{a, 3, 7, 9\}$

- $A' = \{4, 5, 8, b\}$
- $A \cup A' = \{a, 3, 7, 9\} \cup \{4, 5, 8, b\} = \{3, 4, 5, 7, 8, 9, a, b\} = U$
- $A \cap A' = \{a, 3, 7, 9\} \cap \{4, 5, 8, b\} = \emptyset$
- $A' = \{4, 5, 8, b\} \Rightarrow (A')' = \{a, 3, 7, 9\} = A$

Proving (A ∪ B)′ = A′ ∩ B′

De Morgan's Law

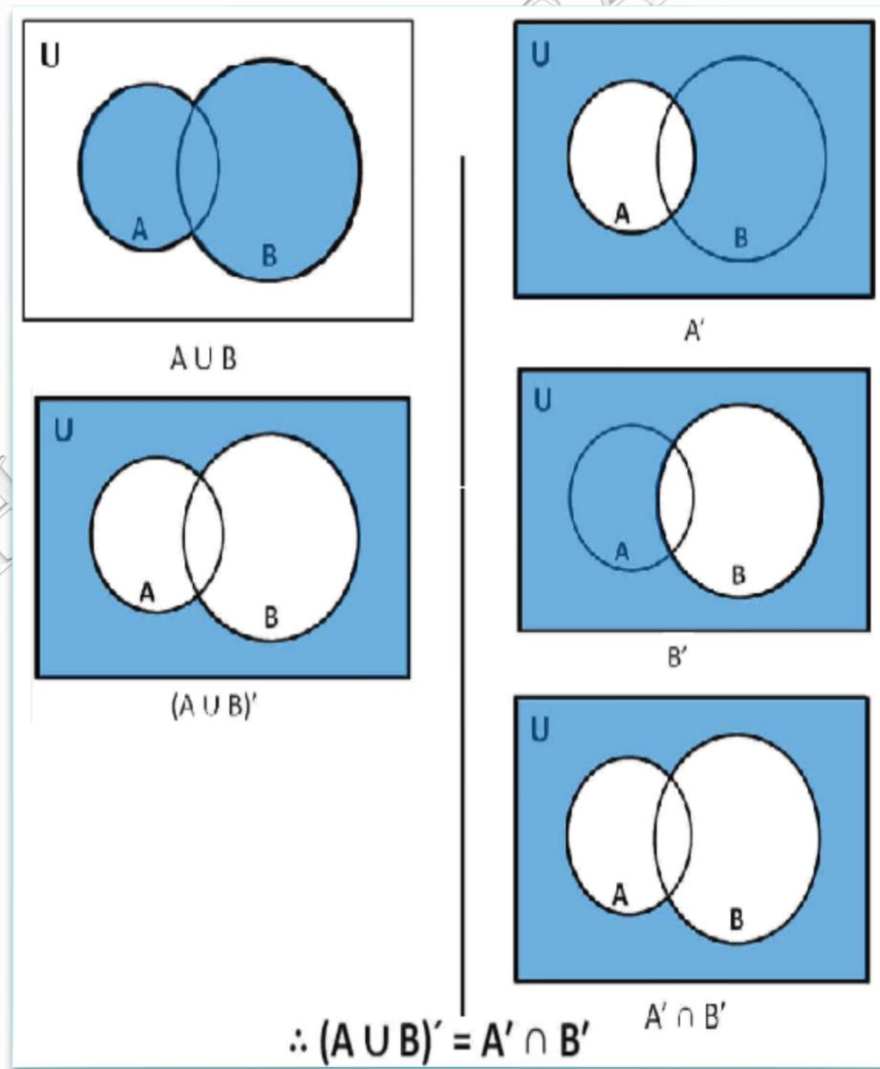
De Morgan's Law are based on complement of sets

1. (A ∪ B)′ = A′ ∩ B′

2. (A ∩ B)′ = A′ ∪ B′

Let us prove the law by Venn Diagrams

(A ∪ B)′ A′ ∩ B′



Lets Try

✡ If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 5\}$,

$B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$ then find:

(i) $(B \cup C)'$

(ii) $(C - A)'$

(iii) $n(A \cup C)$

(iv) $A - (B \cup C)$

(v) $A \cap B$

(vi) $A \cup (B \cap C)$

H.W. EX. 1.5

ASSIGNMENT QUESTIONS

1. Describe the set in Roster form [1]

   $\{x : x \text{ is a two digit number such that the sum of its digit is } 8\}$

2. Are the following pair of sets equal? Give reasons. [1]

   A = { x:x is a letter in the word FOLLOW}

   B = { y:y is a letter in the word WOLF}

$(A - B)'$

4. $A = \{1, 2, \{3, 4\}, 5\}$ which is incorrect and why. (i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ [1]

5. Fill in the blanks. [1]

   (i) $A \cup A' =$-------- (i) $(A \cup B)' =$---------

   (ii) $(A')' =$--------- (ii) $(A \cap B)' =$---------

   (iii) $A \cap A' =$ --------

6. List all the element of the set $A = \{x : x \text{ is an integer } x^2 \leq 4\}$ [1]

7. From the sets given below pair the equivalent sets. [1]

   $A = \{1, 2, 3\}$, $B = \{x, y, z, t\}$, $C = \{a, b, c\}$ $D = \{0, a\}$

8. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ [1]

   Find $(A \cap B) \cap (B \cup C)$

9. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form. [1]

10. Write down all the subsets of the set {1,2,3} [1]

11. Write down all possible proper subsets of the set $\{1, \{2\}\}$. [1]

12. State whether each of the following statement is true or false. [1]

    (i) {2, 3, 4, 5} and {3, 6} are disjoint

    (ii) {2, 6, 10} and {3, 7, 11} are disjoint sets

13. Write the following as interval [1]

    (i) $\{x : x \in R, -4 < x \leq 6\}$

    (ii) $\{x : x \in R, 3 \leq x \leq 4\}$

CONTINUED…

14. If $U = \{a, e, i, o, u\}$ $A = \{a, e, i\}$ $B = \{e, o, u\}$ And $C = \{a, i, u\}$ [4]

Then verity that $A \cap (B - C) = (A \cap B) - (A \cap C)$

15. A and B are two sets such that $n(A - B) = 20 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x + 1$. [4]

Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, Find

(i) the value of x (ii) $n(A \cup B)$

16. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not [4]

coffee, each member drinks tea or coffee.

Find (i) how many drink coffee,

(ii) How many drink coffee, but not tea.

17. If A, B, and C, are three sets and U is the universe set such that $n(U) = 1000$, [4]

$n(A) = 300$, $n(B)$ 300 and $n(A \cap B) = 200$ find $n(A' \cap B')$.

18. In a survey of 60 people, it was found that 25 people read news paper H, 26 [6]

read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H

and T, 8 read both T and I, 3 read all three newspaper. Find

(i) The no. of people who read at least one of the newspapers.

(ii) The no. of people who read exactly one news paper.

19. These are 20 students in a chemistry class and 30 students in a physics class. Find [6]

the number of students which are either in physics class or chemistry class in the following cases.

(i) Two classes meet at the same hour

(ii) The two classes met at different hours and ten students are enrolled in both the courses.

20. In a survey of 25 students, it was found that 15 had taken mathematics, 12had taken [6]

physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects.

Find the no. of students that had taken

(i) only chemistry

(ii) only mathematics

(iii) only physics

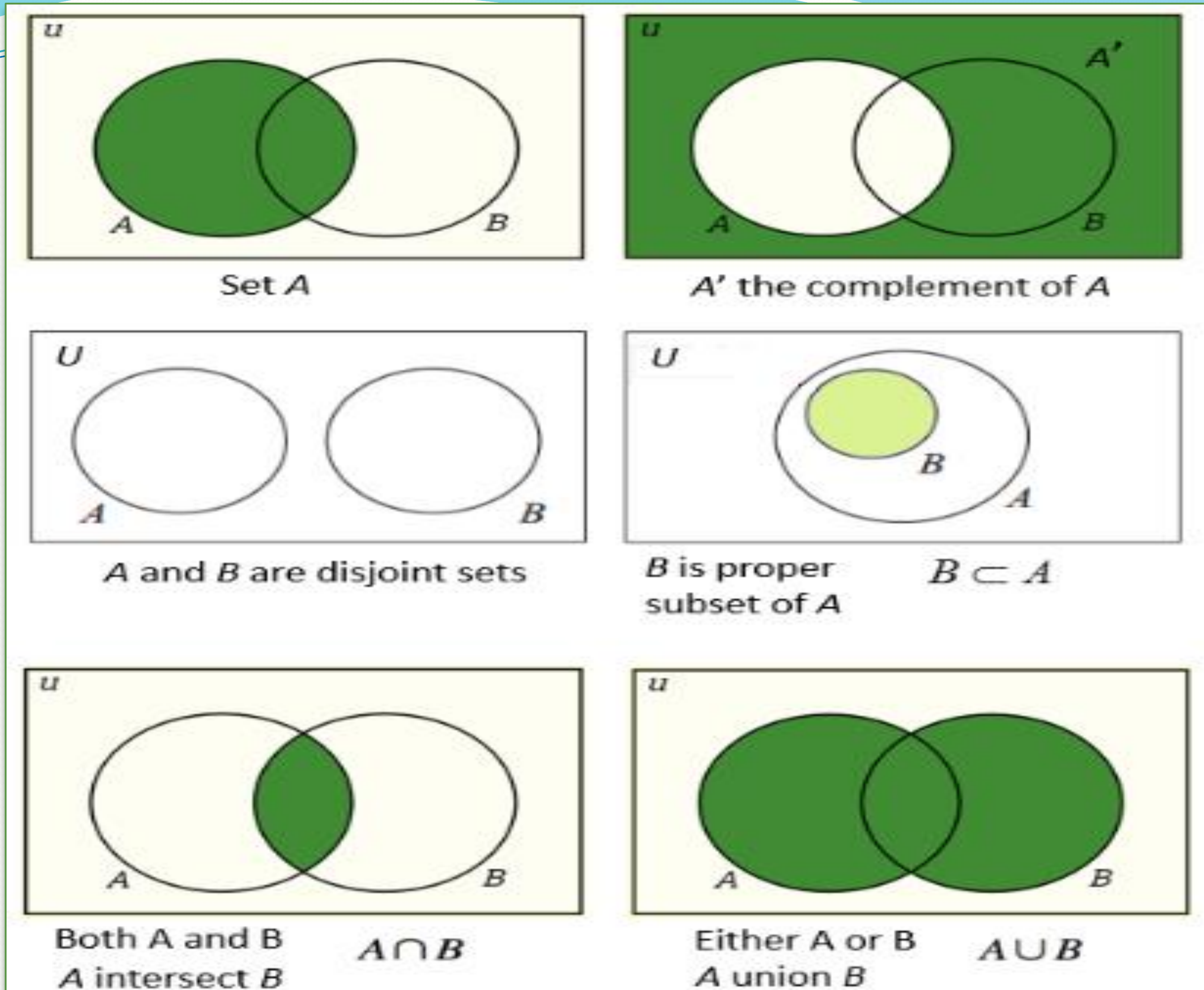
(iv) physics and chemistry but mathematics

(v) mathematics and physics but not chemistry

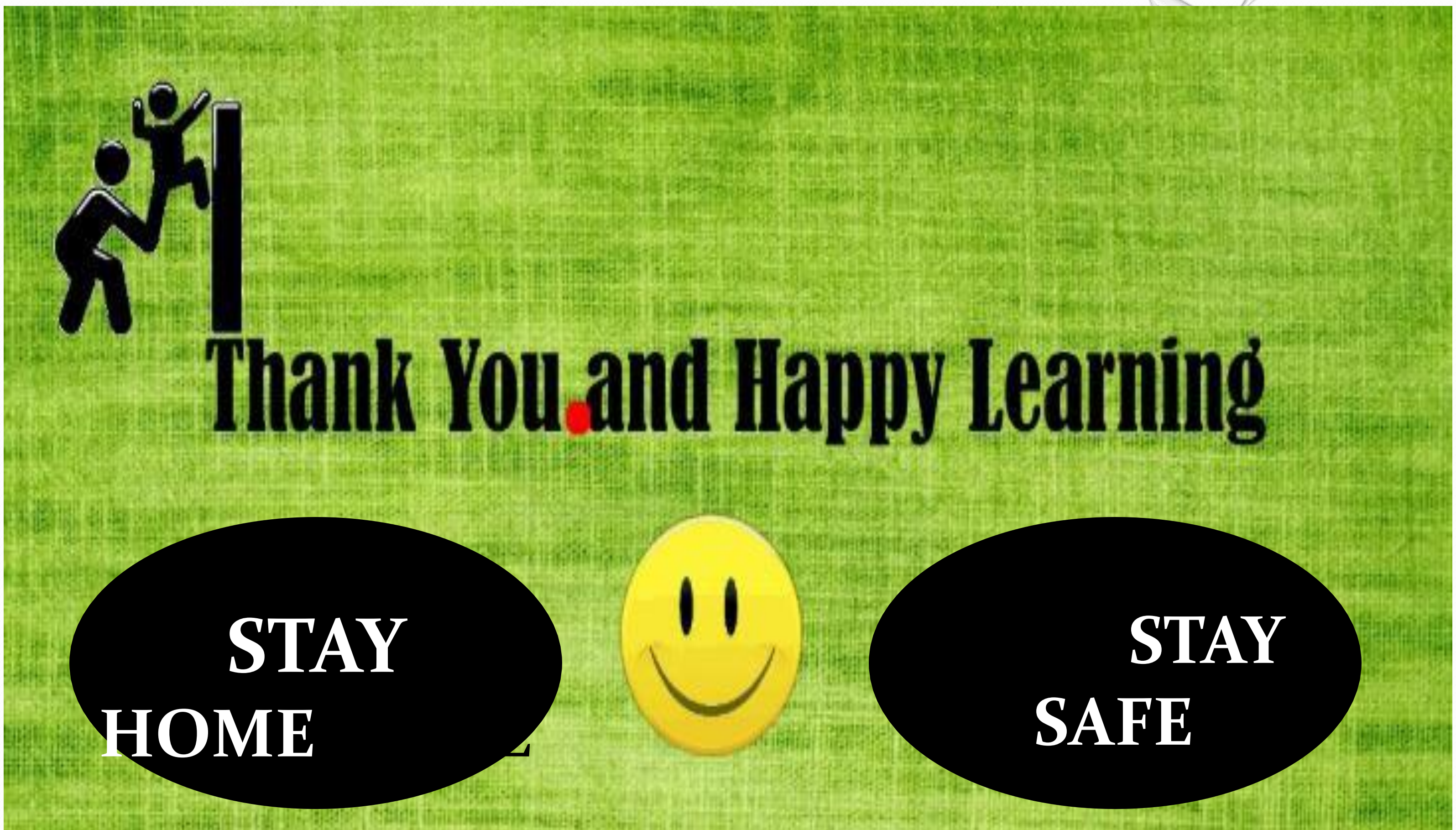
(vi) only one of the subjects

(vii) at least one of three subjects

(viii) None of three subjects.


u
A
B
Set A
u
A'
A
B
A' the complement of A
U
A
B
A and B are disjoint sets
U
B
A
B is proper subset of A
$B \subset A$
u
A
B
Both A and B
A intersect B
$A \cap B$
u
A
B
Either A or B
A union B
$A \cup B$

THANK YOU


Thank You and Happy Learning
STAY
HOME
STAY
SAFE