Selection of a committee of 3 from 10 people

Arrangement of 12 people around a table

Permutations

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

Combinations

$$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

CLASS 11
MODULE 7

A Permutation is an arrangement in a definite order of number of objects taken some or all at a time

$$P(n,r) = \frac{n!}{(n-r)!}$$

RECAP

DIFFERENCES BETWEEN PERMUTATIONS AND COMBINATIONS

PERMUTATIONS	COMBINATIONS
Arranging people, digits, numbers, alphabets, letters, colours.	Selection of menu, food, clothes, subjects, teams.
Keywords: Arrangements, arrange,...	Keywords: Select, choice,...

Combinations

A **Combination** is an arrangement of items in which order does not matter & Repetition is NOT allowed.

To find the number of Combinations of n items chosen r at a time:

$${}_nC_r = C(n,r) = \frac{n!}{r!(n-r)!}$$

Probability Using Permutations and Combinations

- Compute probabilities using combinations.
- Compute probabilities using permutations.

Events with Large Possibilities

When the number of possibilities gets larger, the combination and permutation will be our best friends.

Our general game plan will be to use these rules to find the number of outcomes that satisfy a certain event, as well as the total number of outcomes in the sample space.

Then we can divide the first number by the second to obtain the probability of the event occurring.

EXAMPLE 1 Using Combinations to Compute Probability

Stacy has the option of selecting three books to read for a humanities course. The suggested book list consists of 10 biographies and five current events books. She decides to select the three books at random. Find the probability that all three books selected will be current events books.

SOLUTION

The probability of selecting three current events books is $= \dfrac{{}^{5}C_3}{{}^{15}C_3}$

The probability of selecting three current events books is =

$$\frac{5C_3}{15C_3} = \frac{\dfrac{5.4.3.}{\cancel{3.2.1}}}{\dfrac{15.14.13}{\cancel{3.2.1}}}$$

$$= \frac{10}{455} = \frac{2}{91}$$

EXAMPLE 2 Using Combinations to Compute Probability

What is the probability of getting 4 aces when drawing 5 cards from a standard deck of 52 cards?

$$\frac{4C_4 \times 48C_1}{52C_5} = \frac{1 \times \cancel{48}}{\dfrac{52.51.50.49.\cancel{48}}{5.4.3.2.1}}$$

The probability of getting tour aces is $\dfrac{\boxed{4C_4 \times 48C_1}}{\boxed{{}^{52}C_5}}$

The probability of getting tour aces is $= \dfrac{1}{54{,}145}$

EXAMPLE 3 Using Permutations to Compute Probability

A combination lock has 40 numbers on it, from zero to 39. Find the probability that if the combination to unlock it consists of three numbers, it will contain the numbers 1, 2, and 3 in some order. Assume that numbers cannot be repeated in the combination.

NOTE

SOLUTION

This is a permutation since the order of the numbers is important when you are unlocking the lock.

The probability of the combination containing 1, 2, and 3 is $\frac{{}^{3}P_{3}}{{}^{40}P_{3}}$

$$\frac{3P_3}{40P_3} = \frac{\frac{3!}{0!}}{\frac{40!}{37!}} = \frac{6}{\frac{\cancel{37}!.38.39.40}{\cancel{37}!}}$$

$$= \boxed{\frac{6}{59,280} = \frac{1}{9,880}}$$

EXAMPLE 4 Using Combinations to Compute Probability

A store has six different fitness magazines and three different news magazines. If a customer buys three magazines at random, find the probability that the customer will pick two fitness magazines and one news magazine.

SOLUTION

The probability of selecting two fitness magazines and one news magazine is

$$\frac{{}^{6}C_{2} \times {}^{3}C_{1}}{{}^{9}C_{3}}$$

$$\frac{6C_2 \times 3C_1}{9C_3} = \frac{\frac{6.5 \times 3}{2}}{\frac{9.8.7}{3.2.1}}$$

$$= \frac{45}{84} = \frac{15}{28}$$

Ex 16.3

11) In a lottery, person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by lottery committee, he wins the price. What is the probability of winning the price in the games? [Hint: order of the numbers is not important.]

$\therefore$ Required probability of winning the prize in the game $= \dfrac{1}{{}^{20}C_6}$

TRY

Total number of ways in which one can choose six different numbers from 1 to 20, $= {}^{20}C_6 = 38760$

Hence, there are 38760 combinations of 6 numbers.

Out of these combinations, one combination is already fixed by the lottery committee.

$\therefore$ Required probability of winning the prize in the game $= \dfrac{1}{38760}$

Ex 16.3

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing the Hindi examination?

Let A be the event that the student passes English examination
B be the event that the students passes Hindi examination.

$$P(A \cap B) = 0.5,$$

$$P(\bar{A} \cap \bar{B}) = 0.1$$

$$P(A) = 0.75$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$\boxed{P(E^C \cap F^C) = P((E \cup F)^C) = 1 - P(E \cup F)}$$

$$0.1 = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 0.9$$

TRY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.9 = 0.75 + P(B) - 0.5$$

$$P(B) = 0.65$$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that:

(i) The student opted for NCC or NSS

(ii) The student has opted neither NCC nor NSS

(iii) The student has opted NSS but not NCC

Ex 16.3

Ans. Given: Total number of students $n(S) = 60$

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

$n(A) = 30, n(B) = 32$ and $n(A \cap B) = 24$

$$P(A) = \frac{n(A)}{n(S)} = \frac{30}{60} = \frac{1}{2} \qquad P(B) = \frac{n(B)}{n(S)} = \frac{32}{60} = \frac{8}{15} \qquad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{24}{60} = \frac{2}{5}$$

(i) P(Student opted for NCC or NSS) = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{8}{15} - \frac{2}{5}$$

$$= \frac{19}{30}$$

TRY

(ii) The student has opted neither NCC nor NSS

$$P(\text{Student has opted neither NCC nor NSS}) = P(\bar{A} \cap \bar{B})$$

$$= P(\overline{A \cap B})$$

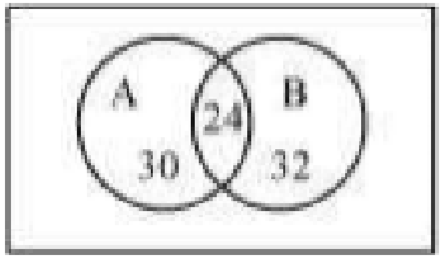
$$= 1 - P(A \cup B)$$

$$= 1 - \frac{19}{30}$$

$$= \frac{11}{30}$$

(iii) The student has opted NSS but not NCC

The given information can be represented by a Venn diagram as



Number of students who have opted for NSS but not NCC $= n(B - A)$

$$= n(B) - n(A \cap B)$$

$$= 32 - 24$$

$$= 8$$

Thus, the probability that the selected student has opted for NSS but not for NCC $= \frac{8}{60} = \frac{2}{15}$

MISCELLANEOUS EXERCISE

Question 2: 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Solution

Number of ways of drawing 4 cards from 52 cards = ${}^{52}C_4$

$\therefore$ Number of ways of drawing 3 diamonds and one spade = ${}^{13}C_3 \times {}^{13}C_1$

Thus, the probability of obtaining 3 diamonds and one spade = $\dfrac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

MISCELLANEOUS EXERCISE

Question 4: In a certain lottery, 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets? (c) 10 tickets?

Total number of tickets sold = 10,000

Number of prizes awarded = 10

(a) If we buy one ticket, then

$$P\text{ (getting a prize)} = \frac{10}{10000} = \frac{1}{1000}$$

$$\therefore P\text{ (not getting a prize)} = 1 - \frac{1}{1000} = \frac{999}{1000}$$

(b) If we buy two tickets, then

Number of tickets not awarded = 10,000 – 10 = 9990

$$P\text{ (not getting a prize)} = \frac{{}^{9990}C_2}{{}^{10000}C_2}$$

(c) If we buy 10 tickets, then

$$P\text{ (not getting a prize)} = \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$$

MISCELLANEOUS EXERCISE

Question 10: The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

The number lock has 4 wheels, each labelled with ten digits i.e., from 0 to 9.

Number of ways of selecting 4 different digits out of 10 digits = ${}^{10}C_4$

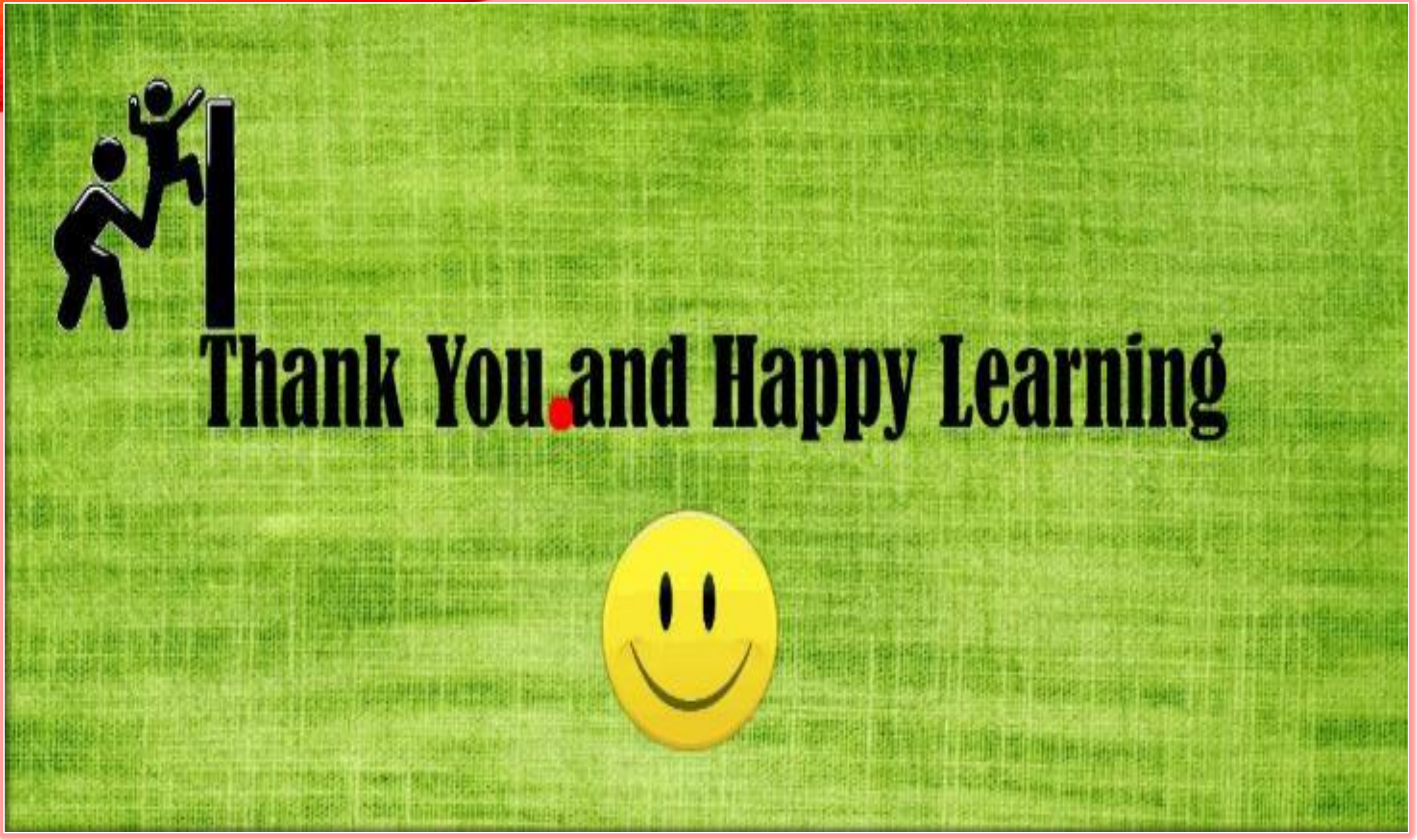
Now, each combination of 4 different digits can be arranged in $4!$ ways.

$\therefore$ Number of four digits with no repetitions = ${}^{10}C_4 \times 4!$

$= 5040$

There is only one number that can be open the suitcase.

Thus, the required probability is $\frac{1}{5040}$.


Thank You and Happy Learning