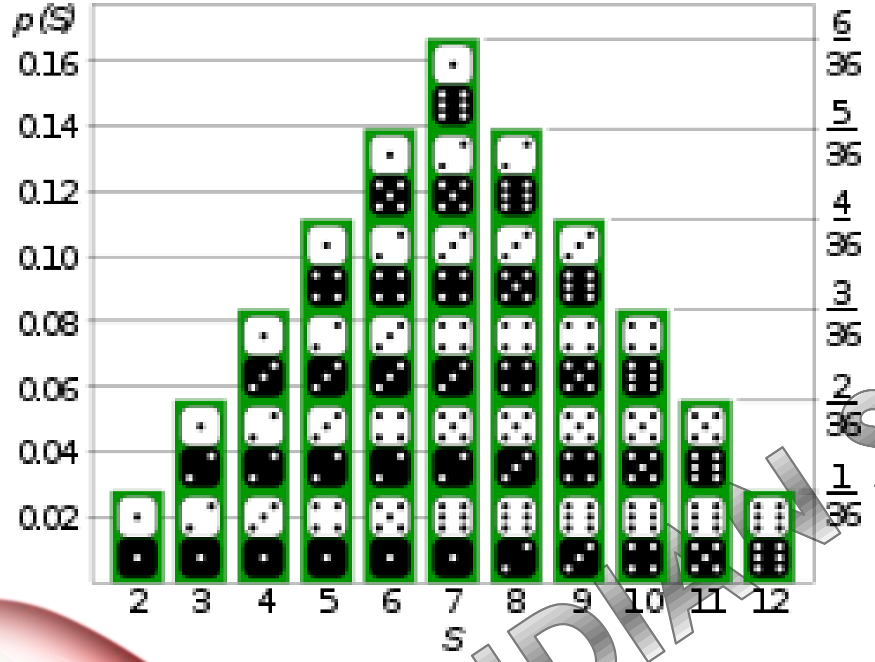
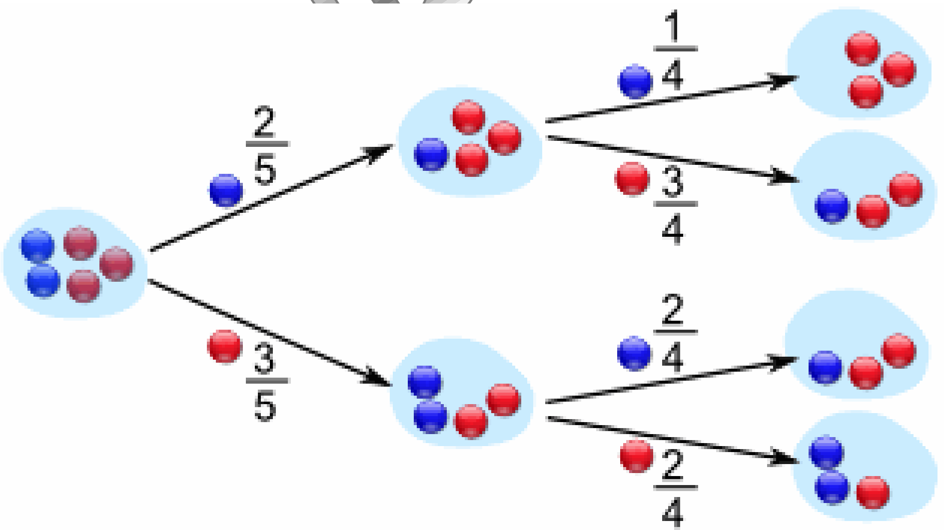
PROBABILITY





CLASS 11
MODULE 5

AXIOMATIC APPROACH TO PROBABILITY

Let S be the sample space of a random experiment and E be an Event associated with S such that $\mathbf{0 \leq P(E) \leq 1}$, then probability P satisfies the following axioms

(i) For any event E, P (E) $\geq$ 0

(ii)P (S) = 1

(iii)If E and F are mutually exclusive events, then

$$P(E \cup F) = P(E) + P(F).$$

RECAP

SOME IMPORTANT RESULTS

RECAP

1)<u>Equally likely outcomes</u>: All outcomes with equal probability

2)<u>Probability of an event</u>: For a finite sample space with equally likely outcomes

$$\text{Probability of an event } P(A) = \frac{n(A)}{n(S)}$$

where n(A) = number of elements in the set A

n(S) = number of elements in the set S.

3)If A and B are any two events, then P(A or B) = P(A) + P(B) – P(A and B)

equivalently, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

RECAP

4)If A and B are mutually exclusive, then P(A or B) = P(A) + P(B)

5)If A is any event, then P(not A) = 1 – P(A)

P(A) + P(not A) = 1

6)

$$P(E^C \cap F^C) = P((E \cup F)^C) = 1 - P(E \cup F)$$

$$P(E^C \cup F^C) = P((E \cap F)^C) = 1 - P(E \cap F)$$

Ex 16.3

3. A die is thrown, find the probability of following events:

(i) A prime number will appear

(ii) A number greater than or equal to 3 will appear

(iii) A number less than or equal to one will appear

(iv) A number more than 6 will appear

(v) A number less than 6 will appear.

Ans.

The sample space of the given experiment is given by

$S = \{1, 2, 3, 4, 5, 6\}$

(i) Let A be the event of the occurrence of a prime number.

Accordingly, $A = \{2, 3, 5\}$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) A number greater than or equal to 3 will appear

(ii) Let B be the event of the occurrence of a number greater than or equal to 3
Accordingly, B = {3, 4, 5, 6}

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(iii) A number less than or equal to one will appear

(iii) Let C be the event of the occurrence of a number less than or equal to one
Accordingly, C = {1}

$$\therefore P(C) = \frac{\text{Number of outcomes favourable to C}}{\text{Total number of possible outcomes}} = \frac{n(C)}{n(S)} = \frac{1}{6}$$

(iv) A number more than 6 will appear

(iv) Let D be the event of the occurrence of a number greater than 6.

Accordingly, D = Φ

$$\therefore P(D) = \frac{\text{Number of outcomes favourable to D}}{\text{Total number of possible outcomes}} = \frac{n(D)}{n(S)} = \frac{0}{6} = 0$$

(v) A number less than 6 will appear.

(v) Let E be the event of the occurrence of a number less than 6.

Accordingly, E = {1, 2, 3, 4, 5}

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{5}{6}$$

Ex 16.3

4. A card is selected from a pack of 52 cards.
(a) How many points are there in the sample space?
(b) Calculate the probability that the card is an ace of spades.
(c) Calculate the probability that the card is
(i) an ace (ii) black card.

(a) There are 52 points in the sample space

(b)

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{1}{52}$$

(c) (i) Let E be the event in which the card drawn is an ace.

Since there are 4 aces in a pack of 52 cards, $n(E) = 4$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(F) = 26$

$$\therefore P(F) = \frac{\text{Number of outcomes favourable to F}}{\text{Total number of possible outcomes}} = \frac{n(F)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Ex 16.3

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Probability of getting sum as 3 $= \frac{1}{12}$

Probability of getting sum as 12 $= \frac{1}{12}$

Result of coin	Result of die	Sum
1	1	2
1	2	3
1	3	4
1	4	5
1	5	6
1	6	7
6	1	7
6	2	8
6	3	9
6	4	10
6	5	11
6	6	12

Ex 16.3

7. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

$$S = \left\{ \begin{array}{c} HHHH,, HHHT, HHTH, HHTT, THHH \\ HTHH, HTHT, HTTH, HTTT, THHT, \\ THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{array} \right\}$$

Total possible outcomes = $2^4 = 16$

Results	Profit on Heads	Loss on tails	Amount of money
4 head 0 tail	$4 \times 1 = 4$	$0 \times 1.5 = 0$	$4 - 0 = 4$
3 head 1 tail	$3 \times 1 = 3$	$1 \times 1.5 = 1.5$	$3 - 1.5 = 1.5$
2 head 2 tail	$2 \times 1 = 2$	$2 \times 1.5 = 3$	$2 - 3 = -1$
1 head 3 tail	$1 \times 1 = 1$	$3 \times 1.5 = 4.5$	$1 - 4.5 = -3.5$
0 head 4 tail	$0 \times 1 = 0$	$4 \times 1.5 = 6$	$0 - 6 = -6$

Results	Amount of money	Various combination possible	Number of outcomes	Probability
4 head 0 tail	4	HHHH	1	$\frac{1}{16}$
3 head 1 tail	1.5	HHHT, HHTH, HTHH, THHH	4	$\frac{4}{16}=\frac{1}{4}$
2 head 2 tail	−1	HHTT, HTHT, HTTH, THHT, THTH, TTHH	6	$\frac{6}{16}=\frac{3}{8}$
1 head 3 tail	−3.5	HTTT, THTT, TTHT, TTTH	4	$\frac{4}{16}=\frac{1}{4}$

TRY FOR 0 Head and 4 Tails

$$\therefore n(S) = 16$$

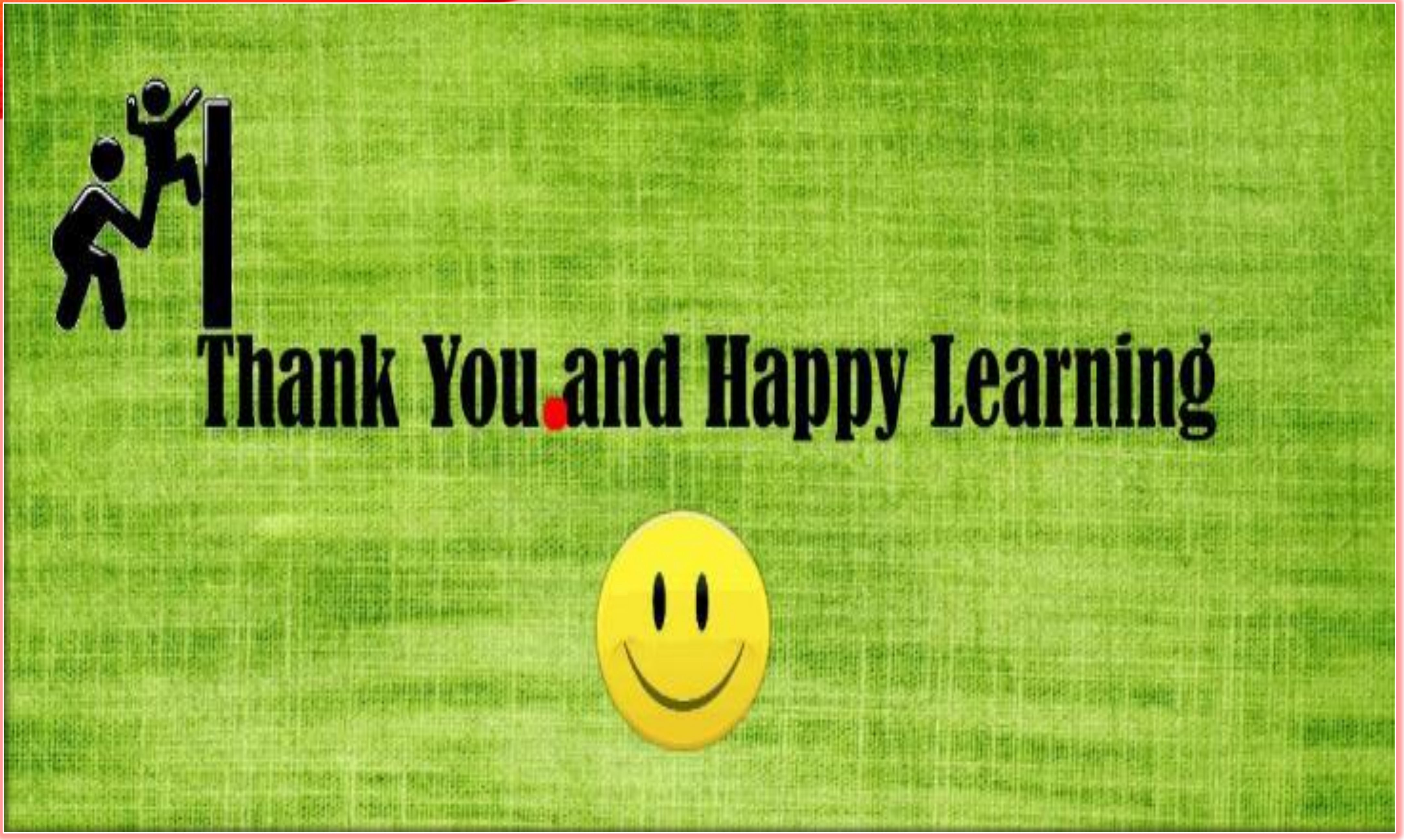
$$\therefore \text{Probability (of winning Rs 4.00)} = \frac{1}{16}$$

$$\therefore \text{Probability (of winning Rs 1.50)} = \frac{4}{16} = \frac{1}{4}$$

$$\therefore \text{Probability (of loosing Re 1.00)} = \frac{6}{16} = \frac{3}{8}$$

$$\therefore \text{Probability (of loosing Rs 3.50)} = \frac{4}{16} = \frac{1}{4}$$

$$\therefore \text{Probability (of loosing Rs 6.00)} = \frac{1}{16}$$


Thank You and Happy Learning