PROBABILITY

CLASS 11
MODULE 4

RECAP

- ***Sample space***: The set of all possible outcomes
- ***Sample points***: Elements of sample space
- ***Event***: A subset of the sample space
- ***Impossible event*** : The empty set
- ***Sure event***: The whole sample space
- ***Complementary event or 'not event'*** : The set A′ or S – A
- ***Event* A *or* B**: The set $A \cup B$
- ***Event* A *and* B**: The set $A \cap B$
- ***Event* A *and* not B**: The set A – B
- ***Mutually exclusive event***: A and B are mutually exclusive if $A \cap B = \phi$
- ***Exhaustive and mutually exclusive events***: Events $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and $E_i \cap E_j = \phi \ \forall\ i \neq j$

In our day to day life we may use words about the chances of occurrence of events. Probability theory attempts to quantify these chances of occurrence or non occurrence of events.

Flipping a Coin

Case (i) A coin is flipped, it either lands on heads (H) or tails (T)

P(head)=1/2 and P(tail) =1/2

Case (ii) when two coins are flipped

In this case there are FOUR possible outcomes. Both coins could land on heads, both could land on tails, one could land on heads and one could land on tails or vice versa.

It's no longer so simple to figure out the probability of any one of these outcomes.

What if you wanted to know the probability that the coins would either both land on heads or both land on tails?

There are some probability rules, or axioms, that you can use to figure this out.

Different approach to probability

RECAP

- ✓ **Statistical approach:** Observation and data collection.
- ✓ **Classical approach:** Only equally likely events.
- ✓ **Axiomatic approach:** For real life events. It closely relates to set theory.

DID YOU KNOW

Axiomatic approach

This approach was proposed by Russian Mathematician A.N.Kolmogorov in1933.

'Axioms' are statements which are reasonably true and are accepted as such, without seeking any proof.

Axiomatic approach is another way of describing probability of an event. In this approach some axioms or rules are depicted to assign probabilities.

AXIOMATIC APPROACH TO PROBABILITY

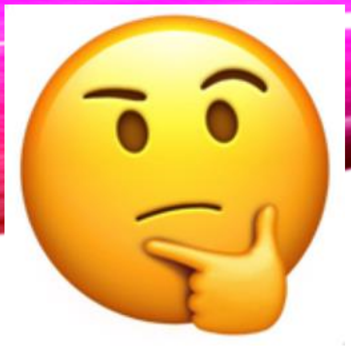
Let S be the sample space of a random experiment and E be an Event associated with S such that $\mathbf{0} \leq \boldsymbol{P}(\boldsymbol{E}) \leq \mathbf{1}$, then probability P satisfies the following axioms

(i) For any event E, P (E) $\geq$ 0

(ii) P (S) = 1

(iii) If E and F are mutually exclusive events, then

$$P(E \cup F) = P(E) + P(F).$$



$P(\varphi) = 0$

Let's find

- **PROOF:**
- We know that;

If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$ -------(1)

- Take $F = \varphi$ in(1)

in(1)

- $P(E \cup \varphi) = P(E) + P(\varphi)$
- $P(E) = P(E) + P(\varphi)$
- i.e. $P(\varphi) = 0$.

Ex 16.3

1) Which of the following can not be valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

(a)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.01	0.05	0.03	0.01	0.2	0.6

Solution :

Here, each of the following $p(\omega_i)$ is less than 1.

Sum of probabilities

$= p(\omega_1) + p(\omega_2) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7)$

$= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= 1$

Thus, the assignment is valid.

(i) For any event E, $\mathbf{0 \le P(E) \le 1}$

(ii) $P(S) = 1$

(c)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.2	0.3	0.4	0.5	0.6	0.7

Here, each of the numbers $p(\omega_i)$ is positive and less than 1.

Sum of probabilities

$= p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 \neq 1$ Thus, the assignment is not valid.

(i) For any event E, $\mathbf{0 \leq \boldsymbol{P(E)} \leq 1}$

(ii) $P(S) = 1$

(d).

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
-0.1	0.2	0.3	0.4	-0.2	0.1	0.3

Here, $p(\omega_1)$ and $p(\omega_5)$ are negative.

Hence, the assignment is not valid.

SOME IMPORTANT RESULTS

1) **Equally likely outcomes:** All outcomes with equal probability

2) **Probability of an event:** For a finite sample space with equally likely outcomes

$$\text{Probability of an event } P(A) = \frac{n(A)}{n(S)}$$

where n(A) = number of elements in the set A

n(S) = number of elements in the set S.

3) If A and B are any two events, then P(A or B) = P(A) + P(B) – P(A and B)

equivalently, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4)If A and B are mutually exclusive, then P(A or B) = P(A) + P(B)

5)If A is any event, then P(not A) = 1 – P(A)

P(A) + P(not A) = 1

6)

$$P(E^c \cap F^c) = P\left((E \cup F)^c\right) = 1 - P(E \cup F)$$

$$P(E^c \cup F^c) = P\left((E \cap F)^c\right) = 1 - P(E \cap F)$$

<u>EXAMPLE </u>1)   LET'S PRACTICE

Suppose the probability of owning a house (H) is 47% while the probability of owning a car (C) is 73%. If the probability of owning a house and a car is 28%, find

(i) the probability of owning a house or a car.

(ii) the probability of not owning a house.

(iii) the probability of neither owning a house nor a car.

(iv) the probability of not owning a house and owning a car.

$P(H) = 0.47$

$P(C) = 0.73$

$P(\text{ H and C}) = 0.28$

$P(H \cap C) = 0.28$

(i) the probability of owning a house or a car.

Soln.

$$P(H \cup C) = P(H) + P(C) - P(H \cap C)$$
$$= 0.47 + 0.73 - 0.28$$
$$= 0.92$$

Therefore, the probability of owning a house or a car is 92%.

(ii) the probability of not owning a house.

Soln.

$$P(H^C) = 1 - P(H)$$
$$= 1 - 0.47$$
$$= 0.53$$

Therefore, the probability of not owning a house is 53%.

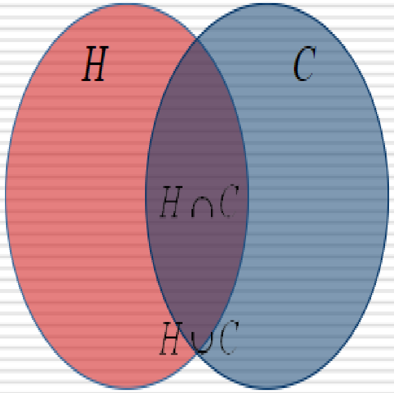
(iii) the probability of neither owning a house nor a car.

(iv) the probability of not owning a house and owning a car.

Soln. We want to find $P\left(H^C \cap C^C\right)$, that is no house and no car.

$$P\left(H^C \cap C^C\right) = P\left(\left(H \cup C\right)^C\right)$$
$$= 1 - P\left(H \cup C\right)$$
$$= 1 - \left[P(H) + P(C) - P\left(H \cap C\right)\right]$$
$$= 1 - \left[0.47 + 0.73 - 0.28\right]$$
$$= 1 - 0.92$$
$$= 0.08$$

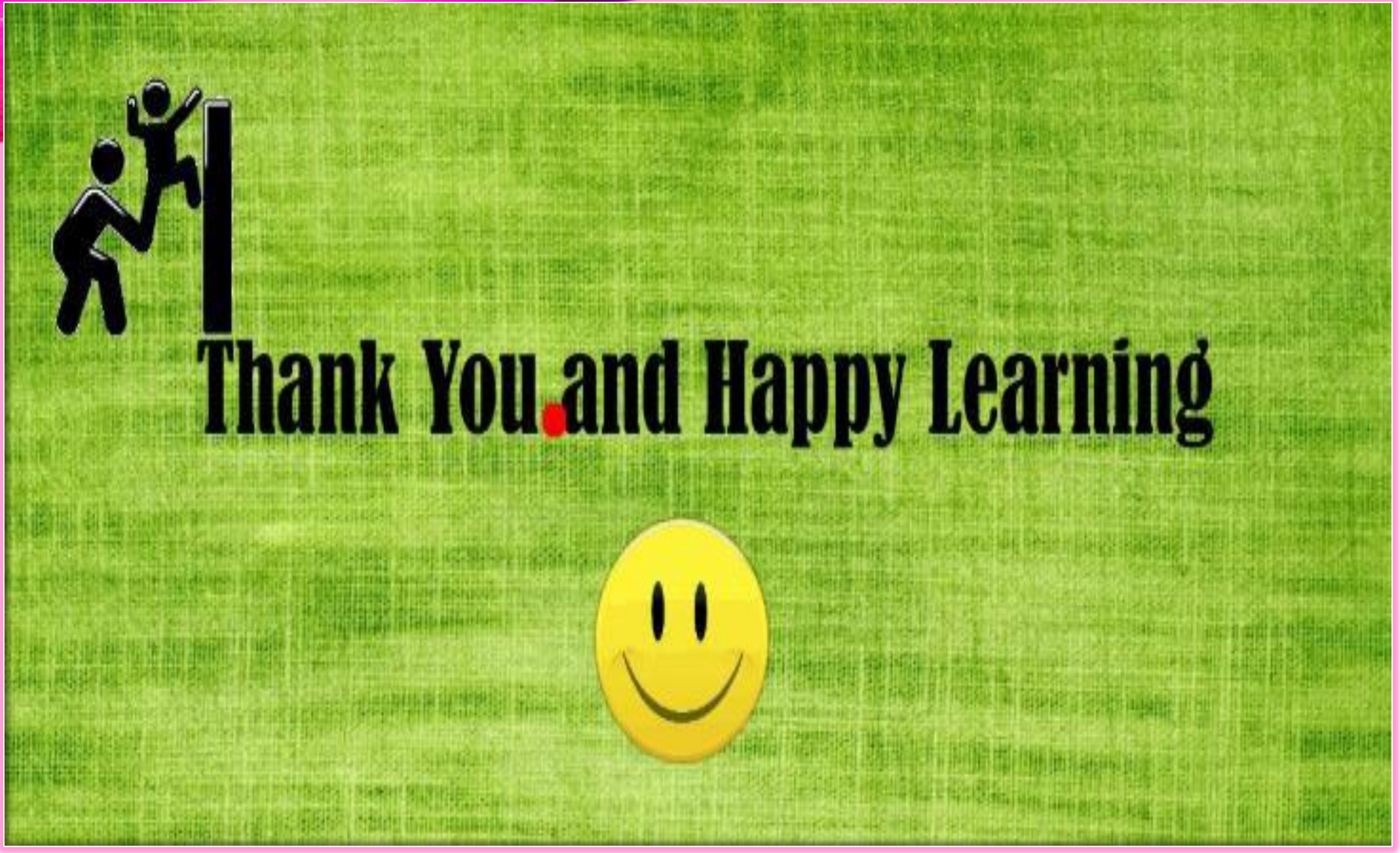
Soln. We want to find $P\left(H^C \cap C\right)$, that is no house and a car. When you want to find "not *A* intersect *B*," draw a Venn diagram.



$$P\left(H^C \cap C\right) = P(C) - P\left(H \cap C\right)$$
$$= 0.73 - 0.28$$
$$= 0.45$$

ASSIGNMENTS

1) If $P(A)$ is $\frac{3}{5}$. Find P (not A)
2) A coin is tossed twice, what is the probability that at least one tail occurs?
3) There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a women?
4) If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.
5) An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Find the probability that they are of different colours


Thank You and Happy Learning