AT

PROBABILITY

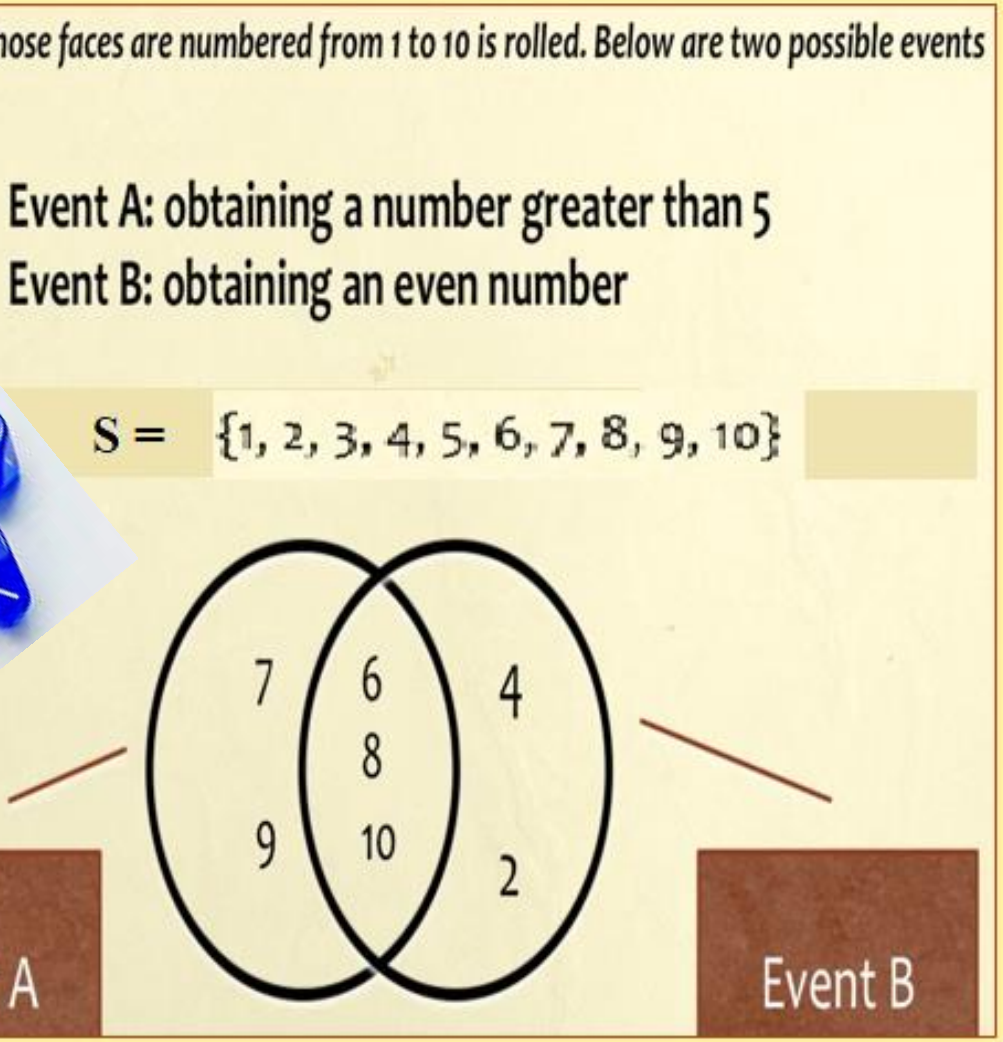
CLASS 11
MODULE 2

Sample space: The set of all possible outcomes

Sample points: Elements of sample space

RECAP

- Any possible outcome of a random experiment is called an event.
- The **probability of an event**, denoted *P*(*E*)
- Example:-
  - Performing an experiment is called trial and outcomes are termed as event.



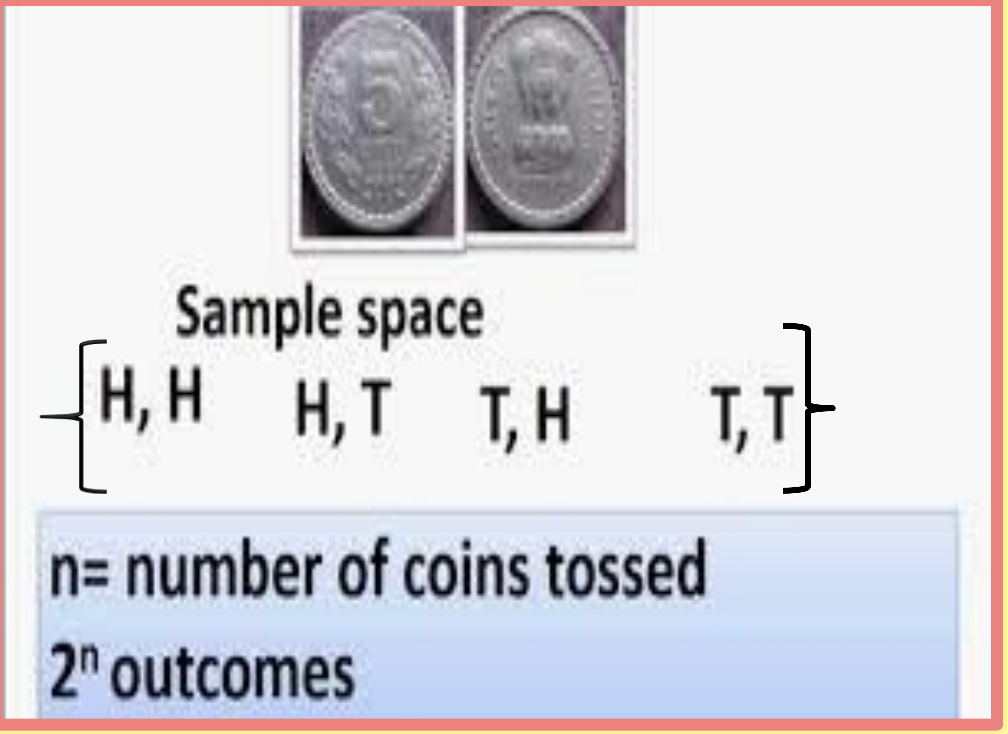
Event

Definition: Any subset E of a sample space S is called an event.

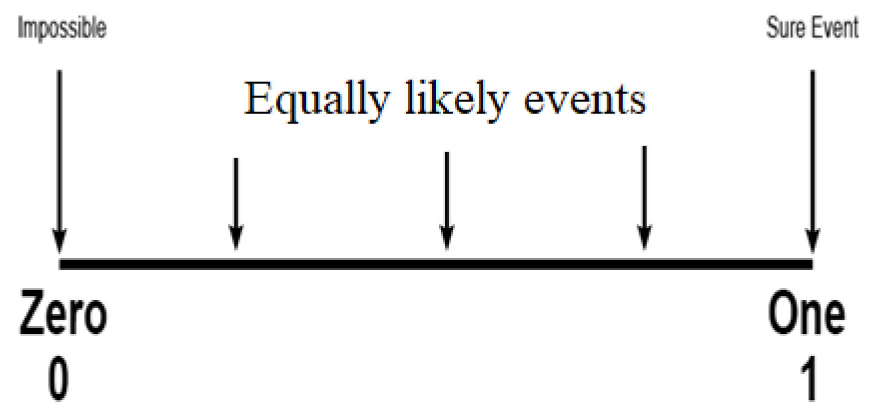
E.g. Tossing a coin two times.

- S = {HH, HT, TH, TT}
- suppose that we are need outcomes of exactly one head.
- E = { HT, TH}

Set E is a subset of the sample space S



TYPES OF EVENTS

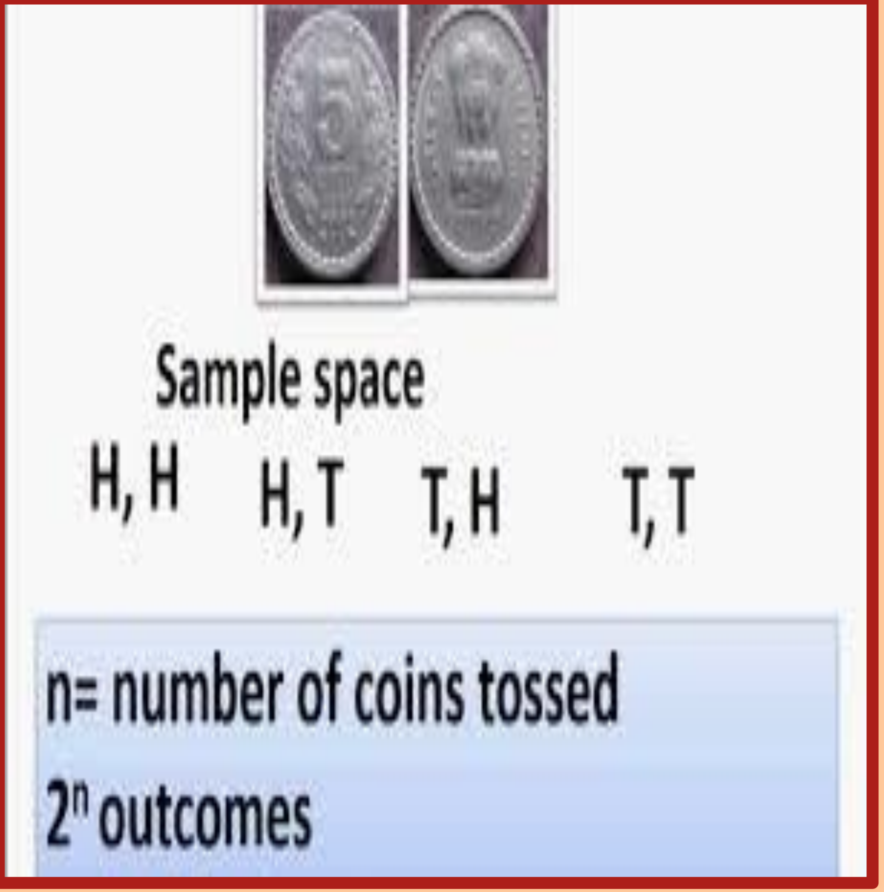


1. **Impossible and Sure Events:** The empty set φ and the sample space S describe events. In fact φ is called an impossible event and S, i.e., the whole sample space is called the sure event.

- For e.g. In the experiment of rolling a die.
- S = {1, 2, 3, 4, 5, 6}
- Let E be the event " the number appears on the die is a multiple of 7".
- Thus, the **event E = φ is an impossible event.**
- Let us take another event F "the number turns up is odd or even"
- F = {1, 2, 3, 4, 5, 6,} = S
- Thus, the **event F = S is a sure event.**

2. <u>Simple Event</u>: If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

- For example in the experiment of tossing two coins
- S={HH, HT, TH, TT}
- There are four simple events corresponding to this sample space.
- These are E1= {HH}, E2={HT}, E3= { TH} and E4={TT}.



3. <u>Compound Event</u>: If an event has more than one sample point, it is called a Compound event.

- For example, in the experiment of "tossing a coin thrice"
- The events E: 'Exactly one head appeared'

  F: 'Atleast one head appeared'

  G: 'Atmost one head appeared' etc. are all compound events.

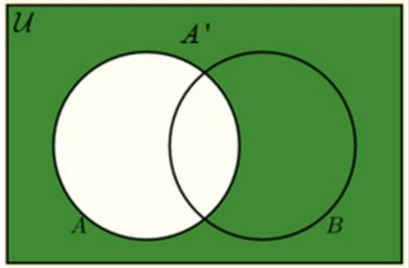
- E={HTT,THT,TTH}

  F={HTT,THT, TTH, HHT, HTH, THH, HHH}

  G= {TTT, THT, HTT, TTH}
- Each of the above subsets contain more than one sample point, hence they are all compound events.

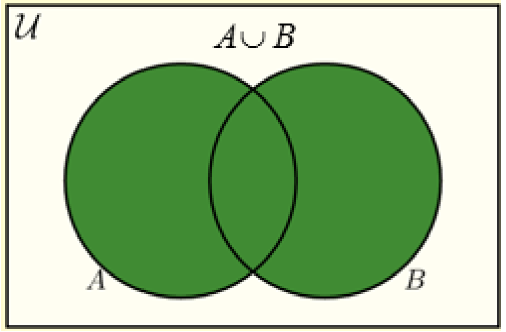
Algebra of events

1. **Complementary Event:** For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A'.

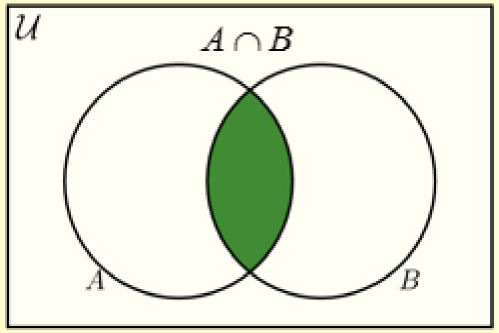


- For example, take the experiment 'of tossing three coins
- S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
- Let A={HTH, HHT, THH} be the event 'only one tail appears'
- Clearly for the outcome HTT, the event A has not occurred.
- we may say that the event 'not A' has occurred.
- Thus, with every outcome which is not in A, we say that 'not A' occurs.
- $A' = \{HHH, HTT, THT, TTH, TTT\}$
- or $A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A$.

2. <u>The Event 'A or B'</u>: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

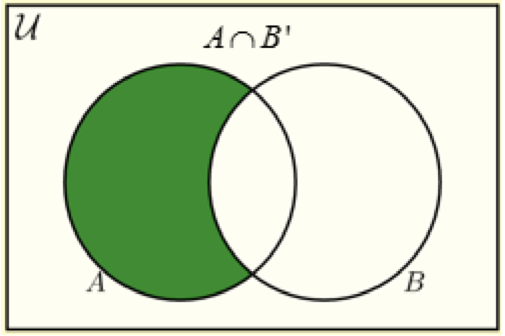


3. <u>The Event 'A and B'</u>: $A \cap B = \{x : x \in A \text{ and } x \in B\}$



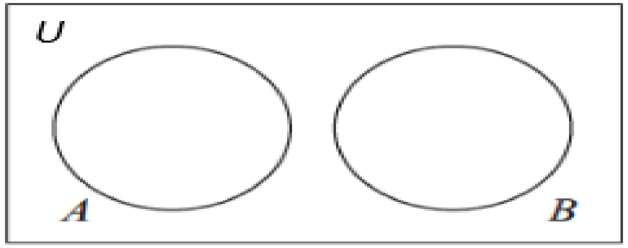
4. <u>The Event 'A but not B'</u>:

$A - B = \{x : x \in A \text{ but } x \notin B\} = A \cap B'$



Mutually Exclusive Events: Two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint.

- For e.g. In the experiment of rolling a die
- S = {1, 2, 3, 4, 5, 6}
- A ‘an odd number appears’ and B ‘an even number appears’
- A = {1, 3, 5} and B = {2, 4, 6}
- Clearly $A \cap B = \varphi$, i.e., A and B are disjoint sets.
- A and B are called mutually exclusive events.

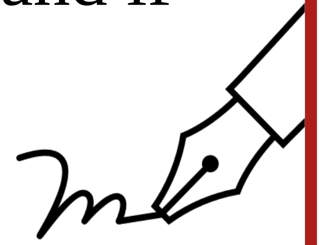


A and *B* are disjoint sets

<u>Exhaustive Events</u>: If $E_1, E_2, \ldots, E_n$ are n events of a sample space S and if

$$E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$$

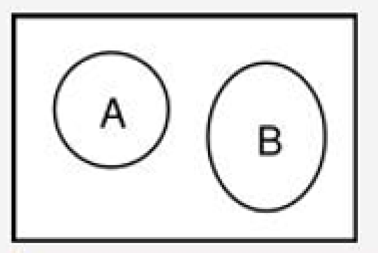
Then $E_1, E_2, \ldots, E_n$ are called exhaustive events.

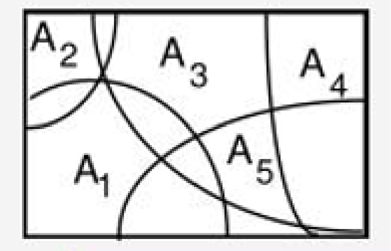


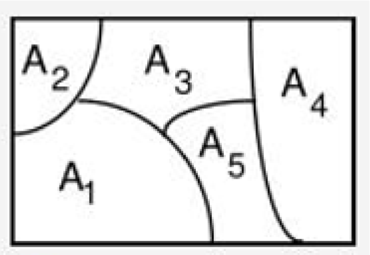
- For e.g. if S = {1, 2, 3, 4, 5, 6}
- Let A: 'a number less than 4 appears'
  B: 'a number greater than 2 but less than 5 appears'
  C: 'a number greater than 4 appears'
- A = {1, 2, 3}, B = {3,4} and C = {5, 6}
- A ∪ B ∪ C = {1, 2, 3} ∪ {3, 4} ∪ {5, 6} = S.
- Such events A, B and C are called exhaustive events.

NOTE:

- Clearly if $E_i \cap E_j = \varphi$, i.e., E_i and E_j are pairwise disjoint and $E_1 \cup E_2 \cup \ldots\ldots\ldots \cup E_n = S$
- Then the events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.







Mutually Exclusive

Collectively Exhaustive

Both Mutually Exclusive and Collectively Exhaustive

Ex16.2, 1

A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{4\}$

$F = \{2, 4, 6\}$

$E \cap F = \{4\} \cap \{2, 4, 6\}$

$= \{4\}$

$E \cap F \neq \phi$

Hence E and F are **not** mutually exclusive events

Ex 16.2, 5

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$A = \{HHH\}$ $\quad$ $B = \{TTT\}$

$A \cap B = \phi$

Hence A & B are mutually exclusive

(ii) Three events which are mutually exclusive and exhaustive

$A = \{HTT, THT, TTH\}$ → exactly two tail comes

$B = \{HHT, HTH, THH, HHH\}$ → at least two head

$C = \{TTT\}$ → only tail

$A \cap B = B \cap C = A \cap C = \phi$

Since A & B, A & C, B & C are mutually exclusive

Hence A, B and C are mutually exclusive

$A \cup B \cup C = S$

Hence A, B & C are exhaustive events

(iii) Two events, which are not mutually exclusive.

$A = \{HHT, HTH, THH, HHH\}$ → at least two head

$B = \{HHH\}$ → only head

$A \cap B = \{HHH\}$

$\neq \phi$ A & B are not mutually exclusive

(iv) Two events which are mutually exclusive but not exhaustive.

$A = \{HHH\}$ $B = \{TTT\}$ $A \cap B = \{HHH\} \cap \{TTT\} = \phi$

A & B are mutually exclusive

$A \cup B = \{HHH\} \cup \{TTT\} \neq S$

A & B are not exhaustive events

(v) Three events which are mutually exclusive but not exhaustive.

TRY

Home work
Ex 16.2 Q no. 6 &7