

**INDIAN SCHOOL MUSCAT**

**SECOND PRE-BOARD EXAMINATION**  
**FEBRUARY 2020**

**SET A**

**CLASS XII  
MATHEMATICS**

**Marking Scheme – SUBJECT [THEORY]**

Q.NO.	Answers								Marks (with split up)		
Q1 to Q20	1. c	2. d	3. d	4. b	5. c	6. b	7. d	8. a	9. c	10. b	Zero or One mark for each question.
11.	x	12.	4	13.	singular	14.	$\left(\frac{3}{4}, \infty\right)$	15.	$\left(\frac{x^2}{2}\right) \log x - \frac{x^2}{4} + c$	16.	$\log x + \sqrt{1+x^2}  + 2\sqrt{1+x^2} + c$
OR				OR						OR	
– 2 cm/min											5/2
17.	proof	18.	$\frac{4}{\sqrt{21}}\hat{i} - \frac{2}{\sqrt{21}}\hat{j} - \frac{1}{\sqrt{21}}\hat{k}$	19.	$p = -2$						
20.	$\frac{1}{4}$										
21.	$= \cot\left[\frac{\pi}{4} - \cot^{-1}\left(\frac{3^2 - 1}{2 \times 3}\right)\right]$			OR	$\tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$					1	
	$= \cot\left(\frac{\pi}{4} - \cot^{-1}\frac{4}{3}\right)$				$2 \cot x \operatorname{cosec} x = 2 \operatorname{cosec} x$						
	$= \frac{\cot\frac{\pi}{4} \cdot \frac{4}{3} + 1}{\frac{4}{3} - \cot\frac{\pi}{4}} = \frac{\frac{4}{3} + 1}{\frac{4}{3} - 1} = 7$				$2 \operatorname{cosec} x(\cot x - 1) = 0$						
					$\operatorname{cosec} x = 0$ is not possible					½	
					$x = \frac{\pi}{4}$						
					(Deduct half mark if $\operatorname{cosec} x = 0$ not rejected)					½	
22.	$\frac{dy}{dx} = -\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x$			$\Rightarrow \frac{d^2y}{dx^2} = -\left[-y \operatorname{cosec} x \cot x + \frac{dy}{dx} \cdot \operatorname{cosec} x\right]$							
	$= -\operatorname{cosec} x (\cot x + \operatorname{cosec} x)$				$= y \operatorname{cosec} x \cot x + y \operatorname{cosec}^2 x$						
	$= -y \operatorname{cosec} x$	(1/2 mark)			$= y \operatorname{cosec} x (\cot x + \operatorname{cosec} x)$	(1/2 mark)					
	$\Rightarrow \frac{d^2y}{dx^2} = y \operatorname{cosec} x \cdot y$			$\Rightarrow \sin x \frac{d^2y}{dx^2} = y^2$							

23.  $y = 2x^3 - 4 \Rightarrow \frac{dy}{dx} = 6x^2$   $\frac{1}{2}$   
 $\therefore \left( \frac{dy}{dx} \right)_{x=2} = 24 \text{ and } \left( \frac{dy}{dx} \right)_{x=-2} = 24$   $\frac{1}{2} + \frac{1}{2}$   
As slopes are equal, therefore tangents are parallel.  $\frac{1}{2}$
24. three vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar  $\frac{1}{2}, \frac{1}{2}$   
 $\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$   $\frac{1}{2}, \frac{1}{2}$   

$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0$$
  
 $\Rightarrow 1(-3 + \lambda) - 3(6 - 0) + 1(2\lambda - 0) = 0$   $\frac{1}{2}, \frac{1}{2}$   
 $\Rightarrow 3\lambda = 21 \Rightarrow \lambda = 7$   
OR  $\vec{r} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = z\hat{j} - y\hat{k}$   $\frac{1}{2}$   
 $\vec{r} \times \hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 0 & 1 & 0 \end{vmatrix} = x\hat{k} - z\hat{i}$   $\frac{1}{2}, \frac{1}{2}$   
 $\therefore (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy = (z\hat{j} - y\hat{k}) \cdot (x\hat{k} - z\hat{i}) + xy$   $\frac{1}{2}$   
 $= 0 - 0 - xy + 0 + xy = 0$   $\frac{1}{2}$
25. It is given that plane is passing through the mid point of line segment joining points A(2, 3, 5) and (4, 5, 7).  
Coordinates of mid point are  $\left( \frac{2+4}{2}, \frac{3+5}{2}, \frac{5+7}{2} \right)$   $\frac{1}{2}$   
i.e., (3, 4, 6)  
D' ratio of line AB are 4 - 2, 5 - 3, 7 - 5 i.e., 2, 2, 2  $\frac{1}{2}$   
∴ Required equation of plane is  $\frac{1}{2}$   
 $2(x-3) + 2(y-4) + 2(z-6) = 0$   $\frac{1}{2}$   
or  $2x + 2y + 2z - 26 = 0$  or  $x + y + z = 13$   $\frac{1}{2}$
26. Required probability =  $P(A)P(\bar{B}) + P(\bar{A})P(B)$   $\frac{1}{2}$   
 $= \frac{60}{100} \times \left( 1 - \frac{90}{100} \right) + \left( 1 - \frac{60}{100} \right) \times \frac{90}{100}$   $\frac{1}{2} + \frac{1}{2}$   
 $= 0.6 \times 0.1 + 0.4 \times 0.9$   
 $= 0.42$   $\frac{1}{2}$

27.

Given that  $(a, b) R (c, d)$  iff  $ad(b+c) = bc(a+d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \quad (1/2)$$

Reflexive. Since  $\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \forall (a, b) \in N \times N$

$\Rightarrow R$  is reflexive.  $\quad (1/2)$

Symmetric. Let  $(a, b) R (c, d)$

$$\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$$

$$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \Rightarrow (c, d) R (a, b)$$

$\Rightarrow R$  is symmetric  $\quad (1)$

Transitive : Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$$\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \text{ and } \frac{1}{c} + \frac{1}{f} = \frac{1}{d} + \frac{1}{e}$$

$$\text{By adding, we get } \frac{1}{a} + \frac{1}{d} + \frac{1}{c} + \frac{1}{f} = \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{f} = \frac{1}{b} + \frac{1}{e} \quad (1)$$

$\Rightarrow (a, b) R (e, f) \Rightarrow R$  is transitive  $(1/2)$

Hence  $R$  is equivalence relation.  $(1/2)$

OR

$$f(x) = \frac{3x+4}{5x-7}$$

$$\text{let } y \in R - \left\{ \frac{3}{5} \right\} : y = \frac{3x+4}{5x-7} \Rightarrow x = \frac{7y+4}{5y-3}$$

1

$$\text{let } g : R - \left\{ \frac{3}{5} \right\} \rightarrow R - \left\{ \frac{7}{5} \right\} \text{ defined as } g(x) = \frac{7x+4}{5x-3}$$

½

showing

$$f \circ g(x) = x \Rightarrow f \circ g(x) = I_{R - \left\{ \frac{3}{5} \right\}} \quad \text{and} \quad g \circ f(x) = x \Rightarrow g \circ f(x) = I_{R - \left\{ \frac{7}{5} \right\}}$$

1 + 1

$$\Rightarrow f \text{ is invertible with } f^{-1} = g \Rightarrow f^{-1}(x) = \frac{7x+4}{5x-3}$$

½

28.

let  $y = u + v$

1/2

$u = x \tan x$

$v = \log \left[ x + \sqrt{x^2 + 1} \right]$

$\log u = \tan x \cdot \log x$

$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \times \left[ 1 + \frac{2x}{2\sqrt{x^2 + 1}} \right]$

1/2, 1

$\frac{du}{dx} = u \left[ \frac{\tan x}{x} + \log x \sec^2 x \right]$

$= \frac{2(x + \sqrt{x^2 + 1})}{(x + \sqrt{x^2 + 1}) \times 2\sqrt{x^2 + 1}}$

1

$\frac{du}{dx} = \left[ \frac{\tan x}{x} + \log x \sec^2 x \right] x \tan x$

$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$

1/2

$\frac{dy}{dx} = \left[ \frac{\tan x}{x} + \log x \sec^2 x \right] x \tan x + \frac{1}{\sqrt{x^2 + 1}}$

1/2

OR

 $\because f(x)$  is continuous at  $x = 1$ .

$\Rightarrow (\text{L.H.L. of } f(x) \text{ at } x = 1) = (\text{R.H.L. of } f(x) \text{ at } x = 1) = f(1)$

$\Rightarrow \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$

1/2

$\text{Now, } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} 5ax + 2b$

$= 5a - 2b$

1

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} 3ax + b$

$= 3a + b$

1

$\text{Also, } f(1) = 11$

Putting these values in (i) we get

$5a - 2b = 3a + b = 11$

$\Rightarrow \begin{aligned} 5a - 2b &= 11 \\ 3a + b &= 11 \end{aligned}$

1/2

On solving (ii) and (iii), we get

$a = 3, b = 2$

1/2 + 1/2

29. Let;  $I = \int (x+3)\sqrt{3-4x-x^2} dx$ .  
 Also let;  $x+3 = p \frac{d}{dx}(3-4x-x^2) + q$

½

$$\Rightarrow x+3 = p(-4-2x) + q$$

$$\Rightarrow x+3 = -4p + q - 2px$$

$$\therefore -2p = 1 \Rightarrow p = -\frac{1}{2}$$

$$\& -4p + q = 3 \Rightarrow q = 1$$

½

$$\therefore I = \int \left\{ -\frac{1}{2}(-4-2x)+1 \right\} \sqrt{3-4x-x^2} dx$$

½

$$\text{put } 3-4x-x^2 = t$$

$$\Rightarrow (-4-2x) dx = dt$$

$$\Rightarrow I = -\frac{1}{2} \int \sqrt{t} dt + \int \sqrt{7-(x+2)^2} dx$$

½ + ½

$$= -\frac{1}{2} \cdot \frac{2}{3} t^{3/2} + \frac{(x+2)}{2} \sqrt{7-(x+2)^2} + \frac{7}{2} \sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$$

½ + ½

$$I = -\frac{1}{3} (3-4x-x^2)^{3/2} + \left(\frac{x+2}{2}\right) \sqrt{3-4x-x^2} + \frac{7}{2} \sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$$

½

30. Let  $x$  units of food P and  $y$  units of food Q are required to be mixed.  
 Cost =  $Z = 4x + 6y$  is to be minimised subject to following constraints.

Constraint  
+ objective

1

$$3x + 6y \geq 80$$

$$4x + 3y \geq 100$$

$$x \geq 0, y \geq 0$$

plotting on graph to get the corner points of the feasible region.

corner points. cost

$$A\left(0, \frac{100}{3}\right) Z|_A = 4 \times 0 + 6 \times \frac{100}{3} = \text{Rs } 200$$

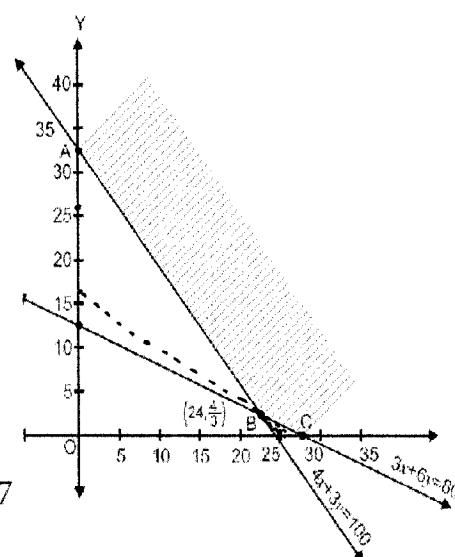
$$B\left(24, \frac{4}{3}\right) Z|_B = 4 \times 24 + 6 \times \frac{4}{3} = \text{Rs } 104$$

$$C\left(\frac{80}{3}, 0\right) Z|_C = 4 \times \frac{80}{3} + 0 = \text{Rs } \frac{320}{3} = \text{Rs } 106.67$$

Thus cost will be minimum if 24 units of P  
 and  $\frac{4}{3}$  units of Q are mixed.

Plot 1

Table 1



The minimum cost is Rs 104.

Verifying  $4x + 6y < 104$  doesn't intersect feasible region.

1

31. Let  $E_1$ ,  $E_2$  and  $A$  be event such that

$E_1$  = Selecting male person  $E_2$  = Selecting women (female person)

½

$A$  = Selecting grey haired person.

$$\text{Then } P(E_1) = \frac{1}{2}, \quad P(E_2) = \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{5}{100}, \quad P\left(\frac{A}{E_2}\right) = \frac{0.25}{100}$$

1

Here, required probability is  $P\left(\frac{E_1}{A}\right)$ .

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$$

½

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2} \times \frac{5}{100}}{\frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{0.25}{100}} = \frac{5}{5 + 0.25} = \frac{500}{525} = \frac{20}{21}$$

1 + 1

OR

$$\therefore \sum_{j=1}^n P_j = 1$$

½

$$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow 10k^2 + 10k - k - 1 = 0 \Rightarrow 10k(k+1) - 1(k+1) = 0$$

$$\Rightarrow (k+1)(10k-1) = 0 \Rightarrow k = -1 \text{ and } k = \frac{1}{10}$$

1

as probability is never negative.

$$\therefore k = \frac{1}{10}$$

½

$$(i) k = \frac{1}{10}$$

$$(iii) P(X > 6) = P(X = 7) = 7k^2 + k$$

½ for each  
correct  
answer.

$$(ii) P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= 7 \times \frac{1}{100} + \frac{1}{10} = \frac{17}{100}$$

$$= 0 + k + 2k = 3k = \frac{3}{10}.$$

$$(iv) P(0 < X < 3) = P(X = 1) + P(X = 2)$$

$$= k + 2k = 3k = \frac{3}{10}.$$

32.  $\Rightarrow \frac{dy}{dx} - \frac{y}{x} + \sin\left(\frac{y}{x}\right) = 0 \quad \dots(i)$

$\frac{1}{2}$

It is homogeneous differential equation.

Let  $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\frac{1}{2}$

Putting these values in (i) we get

$$\begin{aligned} & v + x \frac{dv}{dx} - v + \sin v = 0 \\ \Rightarrow & \frac{dv}{\sin v} = \frac{-dx}{x} \Rightarrow \cosec v dv = -\frac{dx}{x} \end{aligned}$$

1

Integrating both sides we get

$$\begin{aligned} \Rightarrow & \int \cosec v dv = -\int \frac{dx}{x} \\ \Rightarrow & \log \left| \cosec \frac{y}{x} - \cot \frac{y}{x} \right| + \log |x| = c \end{aligned}$$

1

Putting  $x = 2, y = \pi$  we get  $c = \log 2$

$$\begin{aligned} \Rightarrow & \log \left| x \left( \cosec \frac{y}{x} - \cot \frac{y}{x} \right) \right| = \log 2 \\ \Rightarrow & x \left( \cosec \frac{y}{x} - \cot \frac{y}{x} \right) = 2 \end{aligned}$$

1

33. The equation of the plane passing through the line of intersection of the planes

$\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$  is

1

$$\left[ \vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 \right] + \lambda \left[ \vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) \right] = 0$$

$$\Rightarrow \vec{r} \cdot [(1+3\lambda)\hat{i} + (3-\lambda)\hat{j} - 4\lambda\hat{k}] - 6 = 0 \quad \dots(i)$$

$\frac{1}{2}$

$\therefore$  Plane (i) is at unit distance from origin  $(0, 0, 0)$

$$\therefore \left| \frac{0+0-0-6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}} \right| = 1$$

$\frac{1}{2} + 1$

$$\Rightarrow \frac{6}{\sqrt{1+9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2}} = 1$$

$$\Rightarrow \frac{6}{\sqrt{26\lambda^2 + 10}} = 1 \Rightarrow \frac{36}{26\lambda^2 + 10} = 1 \quad [\text{Squaring both sides}]$$

1

$$\Rightarrow 26\lambda^2 + 10 = 36$$

$$\Rightarrow 26\lambda^2 = 26 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

1

Hence, the equations of required planes are

$$\vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) = 6 \quad \text{and} \quad \vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6$$

$\frac{1}{2} + \frac{1}{2}$

34. Let  $R$  be the radius and  $H$  be the height of cone of maximum volume inscribed in a sphere of radius  $r$ .

$$\text{In } \triangle OBD, r^2 = R^2 + (H-r)^2 \Rightarrow R^2 = 2Hr - H^2$$

$$\text{Volume of the cone, } V = \frac{1}{3} \pi R^2 H = \frac{1}{3} \pi H (2Hr - H^2)$$

$$= \frac{2}{3} \pi H^2 r - \frac{\pi H^3}{3}$$

$$\text{Differentiating 'V' w.r. to } H, \frac{dV}{dH} = \frac{4}{3} \pi H r - \pi H^2$$

$$\text{Put } \frac{dV}{dH} = 0, \frac{4}{3} \pi H r - \pi H^2 \Rightarrow H = \frac{4}{3} r$$

$$\Rightarrow \frac{d^2V}{dH^2} = \frac{4}{3} \pi r - 2H\pi \Rightarrow \left. \frac{d^2V}{dH^2} \right|_{H=\frac{4}{3}r} = \frac{4}{3} \pi r - \frac{8}{3} \pi r = -\frac{4}{3} \pi r < 0$$

$$\text{Thus, at } H = \frac{4}{3} r, \text{ volume is maximum and } R^2 = 2\left(\frac{4}{3}r\right)r - \left(\frac{4}{3}r\right)^2 = \frac{8}{9}r^2$$

$$\text{Volume of Cone } V = \frac{1}{3} \pi \left(\frac{8}{9}r^2\right) \left(\frac{4}{3}r\right)$$

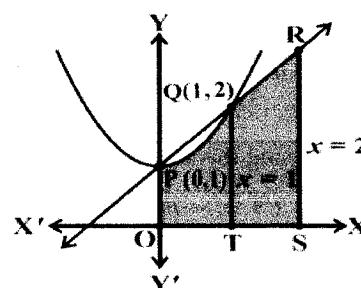
$$\text{Volume of Sphere} = \frac{4}{3} \pi r^3$$

$$\frac{\text{Vol. of Cone}}{\text{Vol. of Sphere}} = \frac{\frac{32}{3} \pi r^3}{\frac{4}{3} \pi r^3} = \frac{8}{27} \Rightarrow \text{Vol. of Cone} = \frac{8}{27} \text{ vol. of Sphere}$$

35.

The points of intersection of

$y = x^2 + 1$  and  $y = x + 1$   
are points  $P(0, 1)$  and  $Q(1, 2)$ .



From the Fig the required region is the shaded region OPQRSTO whose area  
= area of the region OTQPO + area of the region TSRQT

$$= \int_0^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$= \left[ \left( \frac{x^3}{3} + x \right) \right]_0^1 + \left[ \left( \frac{x^2}{2} + x \right) \right]_1^2$$

$$= \left[ \left( \frac{1}{3} + 1 \right) - 0 \right] + \left[ (2 + 2) - \left( \frac{1}{2} + 1 \right) \right] = \frac{23}{6}$$

$$= 23/6 \text{ Sq. units}$$

1

Fig. ½

V in terms of H or r – 1

1

½

1

1

Sketch 1

Points 1

1

1

½ + ½

1

OR

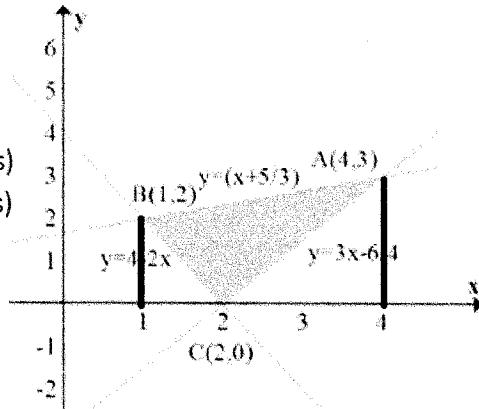
Now, Required area of triangle ABC

= (Area enclosed between the line AB and x-axis)

- (Area enclosed between the line BC and x-axis)

- (Area enclosed between the line AC and x-axis)

Points of intersection  
A(4,3), B(1,2) and C(2,0)



Required area of triangle ABC

$$\begin{aligned}
 &= \left| \int_1^4 \frac{1}{3}(x+5) dx \right| - \left| \int_1^2 (4-2x) dx \right| - \left| \int_2^4 \frac{3}{2}(x-2) dx \right| \\
 &= \frac{1}{3} \left| \left( \frac{x^2}{2} + 5x \right)_1^4 \right| - \left| \left( 4x - \frac{2x^2}{2} \right)_1^2 \right| - \frac{3}{2} \left| \left( \frac{x^2}{2} - 2x \right)_2^4 \right| \\
 &= \frac{1}{3} [8 + 20 - (\frac{1}{2} + 5)] - \{(8 - 4) - (4 - 1)\} - \frac{3}{2} [(8 - 8) - (2 - 4)] \\
 &= \frac{1}{3} (28 - \frac{11}{2}) - (4 - 3) - \frac{3}{2} \times 2 \\
 &= \frac{1}{3} \times \frac{45}{2} - 1 - 3 \\
 &= \frac{15}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units}
 \end{aligned}$$

Plot 1

Points 1

Area st. 1

1

1

1

1

36.  $|A| = 72 - 45 - 99 = -72 \neq 0$

$$\text{adj } A = \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

3

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{-1}{72} \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

$\frac{1}{2}$

$$AX = B \Rightarrow X = A^{-1}B$$

$\frac{1}{2}$

$$\Rightarrow X = \frac{-1}{72} \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix} \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$$

$\frac{1}{2}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{72} \begin{bmatrix} 72 \\ 72 \\ 72 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \Rightarrow \begin{array}{l} x = -1 \\ y = -1 \\ z = -1 \end{array}$$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

OR

$$\text{Let } \Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + p \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} = (1+pxyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad (1/2)$$

Taking  $p$  common from  $C_3$  of second determinant

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + p \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} \quad (1)$$

Taking common  $x, y$  and  $z$  from  $R_1, R_2$  and  $R_3$  respectively of second determinant

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + pxyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad (1)$$

Pass on  $C_3$  over the first two columns of first determinant.

$$\Delta = (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad (1/2)$$

$$\Delta = (1+pxyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} \quad (1)$$

$$= (1+pxyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & (y-x)(y+x) \\ 0 & z-x & (z-x)(z+x) \end{vmatrix}$$

Taking  $(y-x), (z-x)$  common from  $R_2$  and  $R_3$ , we get

$$\Delta = (1+pxyz) (y-x) (z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} \quad (1)$$

Expanding by  $C_1$ , we get

$$\Delta = (1+pxyz) (y-x) (z-x) [z+x-y-x]$$

Hence  $\Delta = (1+pxyz) (x-y) (y-z) (z-x)$  is proved.  $\quad (1)$