

INDIAN SCHOOL MUSCAT

FINAL EXAMINATION

NOVEMBER 2019

SET A

CLASS XII

Marking Scheme – MATHEMATICS

Q.NO .	Answers	Marks (with split up)
1.	(d) $\frac{2}{\sqrt{29}}$ units	1
2.	(b) $\frac{\pi}{4}$	1
3.	(b) 1 , 0 , 0	1
4.	(d) $\frac{2}{9}$	1
5.	(c) $\frac{1}{x}$	1
6.	(c) 3	1
7.	(b) $e^x \sec x + c$	1
8.	(a) $\frac{\pi}{12}$	1
9.	(d) $-\frac{1}{3}$	1
10.	(c) $\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\}$	1
11.	3	1
12.	$6x + 4y + 3z = 12$	1
13.	$E(X^2) - [E(X)]^2$	1
14.	3	1
15.	0	1
16.	$P(A \cap B) = \frac{4}{11} ; P(B/A) = \frac{2}{3}$	$\frac{1}{2} + \frac{1}{2}$
17.	$\tan^{-1} y = \tan^{-1} x + C$	1
18.	a=2	1
19.	$dv = \left(\frac{dv}{dx}\right) \Delta x = 0.06x^3 m^3$	1
20.	(-1 , 1)	1
21.	$x^2 = 4ay$ ----- (i)	Each step

	<p>Differentiating (i) w.r..t x</p> $2x = 4a \left(\frac{dy}{dx} \right) \dots\dots\dots(ii)$ <p>From (i) & (ii) $2x = \frac{x^2}{y} \left(\frac{dy}{dx} \right)$</p> $\Rightarrow x \left(\frac{dy}{dx} \right) - 2y = 0.$	carries $\frac{1}{2}$
22.	<p>$A\vec{B} = \hat{j} + 2\hat{k}$ & $A\vec{C} = \hat{i} + 2\hat{j}$</p> <p>$A\vec{B} \times A\vec{C} = -4\hat{i} + 2\hat{j} - \hat{k}$</p> <p>$A\vec{B} \times A\vec{C} = \sqrt{21}$</p> <p>Area = $\frac{1}{2} A\vec{B} \times A\vec{C} = \frac{1}{2} \sqrt{21}$ sq.units</p> <p>OR</p> <p>The given differential equation is $x \left(\frac{dy}{dx} \right) + 2y = x^2 \dots\dots\dots(i)$</p> <p>Dividing (i) by x, $\left(\frac{dy}{dx} \right) + \frac{2}{x} y = x$</p> <p>I.F = x^2</p> <p>Solution, $y = \frac{x^2}{4} + Cx^{-2}$</p>	Each step carries $\frac{1}{2}$
23.	<p>$\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$</p> <p>$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$</p> <p>$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$</p> <p>$(\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$</p> <p>OR</p> <p>Given, $\vec{a} = 2$, $\vec{b} = 3$ and $\vec{a} \cdot \vec{b} = 4$</p> $ \vec{a} - \vec{b} ^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$ $= \vec{a} ^2 - 2(\vec{a} \cdot \vec{b}) + \vec{b} ^2 = 5$ $= \vec{a} - \vec{b} = \sqrt{5}$	Each step carries $\frac{1}{2}$

24.	$\vec{a}_1 = \hat{i} + \hat{j} ; \quad \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k} ; \quad \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j}$ $\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{59} \quad \text{& Shortest distance , d} = \frac{10}{\sqrt{59}} \text{ units}$	
25.	<p>Let $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$, $\vec{b} = -2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{c} = 5\hat{i} + 3\hat{j} - 3\hat{k}$</p> $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$ $\vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 56$ $\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7, \text{ which is the required equation}$	
26.	<p>Given: $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$</p> $\Rightarrow P(\bar{A}) = \frac{1}{2}, \quad P(\bar{B}) = \frac{2}{3}$ <p>(i) $P(\text{problem is solved}) = 1 - P(\text{none solves})$</p> $= 1 - P(\bar{A}\bar{B}) = \frac{2}{3}$ <p>(ii) $P(\text{exactly one of them solves the problem}) = P(A)P(\bar{B}) + P(B)P(\bar{A}) = \frac{1}{2}$</p>	
27.	$x^2 dy + (xy + y^2) dx = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \frac{y^2}{x^2} \quad \text{-----(I)}$ <p>Put $y = vx \quad \text{-----(II)}$</p> $\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{-----(III)}$ <p>From (i), (ii), (iii) & separating the variables</p> $\frac{dv}{v(v+2)} = -\frac{dx}{x}$ <p>Let $\frac{1}{v(v+2)} = \frac{A}{v} + \frac{B}{v+2}$</p> $= \frac{1}{2v} - \frac{1}{2(v+2)}$ $\frac{1}{2} \left[\frac{1}{v} - \frac{1}{v+2} \right] dv = -\frac{dx}{x}$ <p>Integrating both sides w.r.t.x,</p>	

	$\frac{x^2y}{(y+2x)} = C^2$, which is the general solution	
28	<p>Let A be the event that insured person meets with an accident</p> <p>$P(S) = \frac{1}{6}$, $P(C) = \frac{1}{3}$, $P(T) = \frac{1}{2}$,</p> <p>$P(A/S) = 0.01$ $P(A/C) = 0.03$ $P(A/T) = 0.15$</p> <p>By Bayes' Theorem, $P(S/A) = \frac{1}{52}$</p> <p>OR</p> <p>$P(X=0) = \frac{188}{221}$, $P(X=1) = \frac{32}{221}$ $P(X=2) = \frac{1}{221}$</p> <p>Mean = $E(X) = \frac{34}{221}$, $E(X^2) = \frac{36}{221}$</p> <p>VarX = $\frac{6800}{221 \times 221}$, S.D = 0.37</p>	
29	$\frac{x}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$ $= \frac{\frac{1}{2}}{(x-1)} + \frac{\frac{-1}{2}x + \frac{1}{2}}{x^2+1}$ $\int \frac{x}{(x-1)(x^2+1)} dx = \frac{1}{2} \log x-1 - \frac{1}{4} \log x^2+1 + \frac{1}{2} \tan^{-1} x + C$	
30	$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ ----- (i) $= \int_0^{\frac{\pi}{4}} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$ ----- (by P_4) $\text{On simplification, } I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1 + \tan x}\right) dx$ ----- (ii) Adding (i) & (ii), we get $I = \frac{\pi}{8} \log 2$	
31	$f(x) = 4x^3 - 6x^2 - 72x + 30$ $f'(x) = 12x^2 - 12x - 72$	

	$= 12(x-3)(x+2)$ $f'(x) = 0 \Rightarrow x = -2, x = 3$ f is strictly increasing in the interval $(-\infty, -2) \cup (3, \infty)$ f is strictly decreasing in the interval $(-2, 3)$	
32	$x = a(\cos t + t \sin t) \quad \& \quad y = a(\sin t - t \cos t)$ $\frac{dx}{dt} = at \cos t \quad \& \quad \frac{dy}{dt} = at \sin t$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$ $= \tan t$ $\frac{d^2y}{dx^2} = \sec^2 t \left(\frac{dt}{dx} \right)$ $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}$ <p style="text-align: center;">SECTION:D</p>	
33	Equation of the plane through the line of intersection of the given planes is $[\vec{r}, (\hat{i} + 2\hat{j} + 3\hat{k}) - 4] + \lambda [\vec{r}, (2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$ $\Rightarrow \vec{r} \cdot [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}] + (5\lambda - 4) = 0 \quad \text{(i)}$ (i) Is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0 \quad \text{(ii)}$ On simplification, $\lambda = \frac{7}{19}$ Equation of the required plane is $\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$ <i>i.e</i> $33x + 45y + 50z - 41 = 0$ OR Given equation of the line: $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad \text{(i)}$ General point on the line is $(2 + 3\lambda, 4\lambda - 1, 2 + 2\lambda) \quad \text{(ii)}$ (ii) lies on the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ On simplification, $\lambda = 0$ Point of intersection of the given line & plane is (2, -1, 2) d=13 units	
34	Equation of the circle $x^2 + y^2 = 8x \quad \text{(i)}$ Equation of the parabola $y^2 = 4x \quad \text{(ii)}$ Point of intersection (0, 0) and (4, 4) Area= $\int_0^4 y dx + \int_4^8 y dx$	

$$\begin{aligned}
 &= 2 \int_0^4 \sqrt{x} dx + \int_4^8 \sqrt{4^2 - (x-4)^2} dx \\
 &= \frac{4}{3}(8+3\pi)
 \end{aligned}$$

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$$\begin{aligned}
 f(x) &= 2x^3 - 15x^2 + 36x + 1 \\
 f'(x) &= 6x^2 - 30x + 36 \\
 &= 6(x-3)(x-2) \\
 f'(x) = 0 &\Rightarrow x = 2, x = 3
 \end{aligned}$$

Evaluate the value of f at $x = 1, x = 2, x = 3, x = 5$

$$f(1) = 24, \quad f(2) = 29 \quad f(3) = 28 \quad f(5) = 56$$

Absolute maximum value = 56 at x=5

Absolute minimum value = 24 at x=1

OR

Let x be the radius and y be the height of the cylinder which is inscribed in a sphere of radius R .

$$4x^2 + y^2 = 4R^2$$

$$V = \pi x^2 y = \frac{\pi}{4} [4R^2 y - y^3]$$

$$\frac{dv}{dy} = \frac{\pi}{4} [4R^2 - 3y^2]$$

$$\text{For maximum } V, \frac{dv}{dy} = 0$$

$$\Rightarrow \frac{dv}{dy} = [4R^2 - 3y^2] = 0$$

$$\Rightarrow y = \frac{2R}{\sqrt{3}}$$

$$\frac{d^2v}{dy^2} = \frac{-6\pi y}{4}$$

$$\text{At } y = \frac{2R}{\sqrt{3}}, \quad \frac{d^2v}{dy^2} < 0$$

$$\therefore V \text{ is maximum at } y = \frac{2R}{\sqrt{3}}$$

$$V = \frac{4}{3\sqrt{3}} \pi R^3$$

36. **Solution** Suppose x is the number of pieces of Model A and y is the number of pieces of Model B. Then

$$\text{Total profit (in Rs)} = 8000x + 12000y$$

Let

$$Z = 8000x + 12000y$$

We now have the following mathematical model for the given problem.

$$\text{Maximise } Z = 8000x + 12000y \quad \dots (1)$$

subject to the constraints:

$$9x + 12y \leq 180 \quad (\text{Fabricating constraint})$$

i.e.

$$3x + 4y \leq 60 \quad \dots (2)$$

$$x + 3y \leq 30 \quad (\text{Finishing constraint}) \quad \dots (3)$$

$$x \geq 0, y \geq 0 \quad (\text{non-negative constraint}) \quad \dots (4)$$

The feasible region (shaded) OABC determined by the linear inequalities (2) to (4) is shown in the Fig 12.9. Note that the feasible region is bounded.

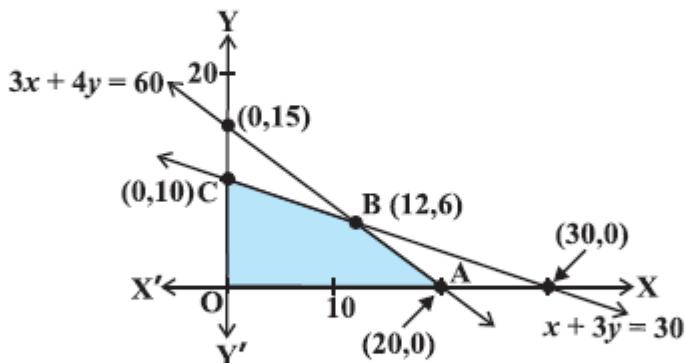


Fig 12.9

Let us evaluate the objective function Z at each corner point as shown below:

Corner Point	$Z = 8000x + 12000y$
O (0, 0)	0
A (20, 0)	160000
B (12, 6)	168000 ←
C (0, 10)	120000

Maximum

We find that maximum value of Z is 1,68,000 at B (12, 6). Hence, the company should produce 12 pieces of Model A and 6 pieces of Model B to realise maximum profit and maximum profit then will be Rs 1,68,000.

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SET:B

$$y = a \sin(x + b) \dots (i)$$

Differentiating (i) w.r.t x

$$\left(\frac{dy}{dx} \right) = a \cos(x + b) \quad \text{--- (ii)}$$

Differentiating (ii) w.r.t x

$$\left(\frac{d^2y}{dx^2} \right) = -a \sin(x + b) \quad \text{--- (iii)}$$

$$\text{From (i), (ii) & (iii)} \left(\frac{d^2y}{dx^2} \right) + y = 0$$

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SET:C

$$\text{Given: } |\vec{a}| = 2, |\vec{b}| = 1 \text{ and } \vec{a} \cdot \vec{b} = \frac{1}{2}$$

$$\begin{aligned} (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) &= 6|\vec{a}|^2 + 11(\vec{a} \cdot \vec{b}) - 35|\vec{b}|^2 \\ &= \frac{-11}{2} \end{aligned}$$