INDIAN SCHOO MUSCAT
QUESTION BANK 2017-18

DEPARTMENT OF MATHEMATICS – SENIOR SECTION

Relations and Functions

1 mark (For conceptual practice)

1. Which of the following graphs of relations defines a transitive relation on
   A = { 1 , 2 , 3 , 4 } ?
   R_1 = { (1 , 2) , (3 , 4) , (2 , 3) , (2 , 4) }
   R_2 = { (1 , 2) , (3 , 4) , (2 , 4) }

2. If f : R $\rightarrow$ R is given by f(x) = x^3 + x , find $f^{-1}(x)$.

3. Let f(x) = $\frac{1}{1-x}$, then what is the value of (fofof)(x)

4. Let f(x) = $\frac{\alpha x}{x+1}$, what is the value of α such that f(f(x)) = x ?

5. Let ∗ be a binary operation on N defined by a ∗ b = a + b + 10 for all a , b ∈ N. write the identity element for ∗ in N.

6. Let ∗ be a binary operation defined on Q^+ by a ∗ b = $\frac{ab}{3}$ for all a .b ∈ Q^+. What is the inverse of 4 ∗ 6 ?

7. If f : R $\rightarrow$ R is given by f(x) = 3x – 5 . Find $f^{-1}(x)$

4 MARK QUESTIONS

8. Prove that the function R on the set Z of all integers defined by: {(x , y) ∈ R $\Leftrightarrow$ x – y is divisible by 4} , is an equivalence relation.

9. Discuss the commutativity and the associativity of the binary operation ∗ on R defined by a ∗ b = $\frac{ab}{4}$, a , b ∈ R.

10. Let L be the set of lines in the XY – plane and R be a relation in L defined as R = { (L_1, L_2) : L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x +4.

11. Show that the function f : Q $\rightarrow$ Q defined by f(x) = 3x + 5 , for all x ∈ Q

is one and onto. Hence or otherwise, find f^{-1}.

12. Let $*$ be a binary operation on N set of , the set of natural numbers defined by $a * b = a^b$, $a, b \in N$. Show that $*$ is neither commutative nor associative.

13. The binary operations $*$ and o on R defined by $a * b = |a - b|$ and $a \circ b = a$

Show that i) $*$ is commutative but not associative
ii) o is associative but not commutative
iii) $*$ is distributive over o

14. Let $f : W \to W$, be defined by $f(n) = \begin{cases} n+1, \text{if } n \text{ is even} \\ n-1, \text{if } n \text{ is odd} \end{cases}$

Show that f is invertible and $f = f^{-1}$

15. Let A be the set of all numbers except -1 and o be an operation defined on A by $aob = a + b + ab$, $a, b \in A$

Prove that i) A is closed under the given operation
ii) o is commutative as well as associative
iii) the number 0 is an identity element
iv) each element a of a has $-\frac{a}{a+1}$ as its inverse

6 Marks Questions

16. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$.

17. Show that $f : R - \{-1\} \to R - \{-1\}$ given by $f(x) = \frac{x}{x+1}$ is invertible.

Also find f^{-1}

18. Let $A = Q \times Q$ and let $*$ be a binary operation defined on A by $(a, b) * (c, d) = (ac, ad + b)$.

i) Is $*$ commutative ?
ii) Is $*$ associative ?
iii) Find the identity element of $(A, *)$, if exists.
iv) Find the invertible elements of $(A, *)$.

19. Let X be a non – empty set. P(X) be its power set. Let $*$ be a binary operation defined on P(X) by $A * B = A \cap B$ for all $A, B \in P(X)$. then prove that

i) $*$ is a binary operation on P(X).

ii) Is $*$ associative ?

iii) Is $*$ commutative ?

iv) Find the identity element in P(X) w.r.t $*$

v) Find all the invertible elements of P(X)

If o is another binary operation defined on P(X) by AoB = A $\cup$ B , then prove that $*$ is distributive over o.

20. Let A = N $\times$ N and let $*$ be a binary operation defined on A by (a, b) $*$ (c , d) = (ad + bc , bd) .

i) Show that $*$ is associative.

ii) Is $*$ commutative?

iii) Show that (A , $*$) has no identity element.

INVERSE TRIGONOMETRIC FUNCTIONS

1marks

1. Write the principal value of $\cos^{-1}(\cos(680^o))$.
2. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.
3. Find the value of $\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$
4. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.
5. Using principal values, Write the value of $\left[\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)\right]$.
6. Evaluate: $\cot^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.
7. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $|x|>1$in the simplest form.
8. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}, xy<1$then write the value of $x + y + xy$.
9. Find the value of $\sec\left(\tan^{-1}\frac{y}{2}\right)$.
10. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$,then find the value of $\cos^{-1}x+\cos^{-1}y$

2Marks

1. Write in simplest form : $\tan^{-1}\left[\frac{cosx}{1+sinx}\right], x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.
2. Show that $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}$.
3. Sow that $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}=\frac{\pi}{4}$.
4. Evaluate $4\tan^{-1}\frac{1}{5}$.
5. Show that $\tan^{-1}\frac{2}{3}=\frac{1}{2}\tan^{-1}\frac{12}{5}$.
6. $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^3}\right)=3\tan^{-1}\left(\frac{x}{a}\right)$.
7. Prove that : $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}, if\ x\in[-1,1]$.

8. Solve the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$.

9. Write in the simplest form $\sin^{-1}\left[\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right], -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.

10. If $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1} x$ then show that $x = \frac{a+b}{1-ab}$

4Marks

1. Solve the equation for x: $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.
2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2}$.
3. If $\sin(\cot^{-1}(x+1)) = \cos(\tan^{-1} x)$, *then find* x.
4. Prove that $\cos(\tan^{-1}\{\sin(\cot^{-1} x)\}) = \sqrt{\frac{1+x^2}{2+x^2}}$.
5. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$.
6. Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$.
7. Solve for x:$\cos[\tan^{-1}(x)] = \sin\left[\cot^{-1}\frac{3}{4}\right]$.
8. Simplify:$\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right], if\ \frac{a}{b}\tan x > -1$.
9. Find the value of the expression $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos\left(\tan^{-1} 2\sqrt{2}\right)$.
10. Solve for x:$2\tan^{-1}(sinx) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$.

MATRICES AND DETERMINANTS

One mark questions

1. If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A = k^2 A , then write the value of k .

2. Construct a 2 x 2 matrix , A = $[a_{ij}]$, whose elements are given by $a_{ij} = \frac{(i+2j)}{2}$

3. Find x from the matrix $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

4. If A = $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$, find B such that 2A – B +3I = O

5. Find the number of all possible matrices of order 2 x 2, with each entry as 1 or 2 .

6. Using determinants, find the area of triangle whose vertices are (2,7), (1,1) and (10,8).

7. For what value of x, the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix.

8. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

9. If A is a non-singular matrix and A^2 = I, then find A^{-1} .

10. If the determinant of the matrix A, of the order 3 x 3 , is 4 then find the value of $|3A|$.

Two mark questions

11. If A and B are symmetric matrices then show that BA – 2AB is neither symmetric nor skew symmetric.

12. Express the matrix A = $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ as sum of symmetric and skew symmetric matrices.

13. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that (A – A') is a skew symmetric matrix.

14. For what value of k , the matrix $\begin{bmatrix} 1 & k & 0 \\ 3 & -1 & 2 \\ 4 & 1 & 5 \end{bmatrix}$ is singular ?

15. Find the inverse of matrix $\begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$.

16. Using elementary row transformation find the inverse of matrix $\begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix}$.

17. Show that the points (a + 5 , a – 4), (a – 2 , a + 3) and (a , a) do not lie on a straight line for any value of 'a'.

15. If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, then find the values of a, b and c.

16. Solve the matrix equation $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

17. If A = diag [1, x] , B = $\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ and A + B = C, find x and y.

18.

FOUR MARK QUESTIONS

19. Use matrix multiplication to divide Rs. 30000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts Rs. 3060.

20. Three schools A, B and C organised a mela for collecting funds for helping rehabilitation of earthquake victims. They sold hand-made fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100 and Rs. 50 each respectively. The number of articles sold by each school is given below

21.

Articles/ Schools	A	B	C
Hand-made fans	40	25	35
Mats	50	40	50
Plates	20	30	40

22. Find the funds collected by each school separately by the above sale. Also find the total fund generated. Write one value depicted in the above situation.

Using elementary row transformation, find the inverse of the matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

Solve the matrix equation $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$.

If A = $\begin{bmatrix} 4 & 0 \\ 2k & 5k \end{bmatrix}$ and B = $\begin{bmatrix} k & 0 \\ 3 & -1 \end{bmatrix}$ such that AB = BA.

23. Then show that $2k^2 + 17k - 12 = 0$.

For questions 26 to 28 without actual expanding show that

24. $\begin{vmatrix} 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \\ a-b-c & 2a & 2a \end{vmatrix}$ = (a + b + c)^3

25. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ = 2abc(a + b + c)^3

26. If $\begin{vmatrix} 4-x & 4+x & 4-x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$, then find the value of x.

27. If f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of f(2x) – f(x) .

Six mark questions

28. Find A^{-1}, where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the following system of linear equations.
x + 2y – 3z = - 4 , 2x + 3y +2z = 2 and 3x – 3y – 4z = 11

29. Find A^{-1}, where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the following system of linear equations
x +2y +3z = 4, 2x + 3y – 3z = -1 and 3x – 2y + 4z = 7.

30. If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Use this answer to solve the following system of linear equations,
x – y + z = 4, x – 2y – 2z = 9 and 2x + y +3z = 1.

31. If A + B + C = 0, then show that $\begin{vmatrix} 1 & cosC & cosB \\ cosC & 1 & cosA \\ cosB & cosA & 1 \end{vmatrix} = 0$

or

If A , B and C are the interior angles of a triangle , then show that $\begin{vmatrix} 1 & cosC & cosB \\ cosC & 1 & cosA \\ cosB & cosA & 1 \end{vmatrix} = 0$

32. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs x each, Rs y each, Rs z each for the three respective values to 3,2 and 1 students respectively with a total award money of Rs 1600. School B wants to spend Rs 2300 to award its 4,1 and 3 students on the respective values(by giving the same award money to the three values as before).If the total amount of award for one prize on each value is Rs 900, using **matrices** , find the award money for each value.

33. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$.

Hence find A^{-1} .

Find the inverse of a matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row operations.

34. Find the value of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A' = A^{-1}.

35. For matrix A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find values of a and b such that $A^2 + aI = bA$

Using properties of determinants, prove the following :

$\begin{vmatrix} a^2 & bc & ac+c^2 \\ ab+a^2 & b^2 & ac \\ ab & bc+b^2 & c^2 \end{vmatrix} = 4\, a^2 b^2 c^2.$

CONTINUITY AND DIFFERENTIABILITY:

1 MARK QUESTIONS :

1. Discuss the continuity of the function $f(x) = \frac{1}{x-5}$ for $x \in R$.

36. 2. Examine the continuity of the function $f(x) = x^2 + 5$ at $x = -1$.

3. Find $\frac{dy}{dx}$, if $x^2 + y^2 = 5$

4. Differentiate $\sin^{-1} x^2$, with respect to x.

37. 5. Find $\frac{dy}{dx}$, if $\sin y + x = \log x$

6. Differentiate e^{-2x}, with respect to x.

7. Differentiate $5^{\sin x}$, with respect to x.

38. 8. If $y = \sin 3x$, find y_2.

9. Find $\frac{d^2y}{dx^2}$, if $y = e^{-3x}$

39. 10. Differentiate $\log(\log x)$, with respect to x.

2 MARK QUESTIONS :

40. 11. For what value of k is the function defined by

$$f(x) = \begin{cases} \frac{\sin x + x\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$ continuous at $x = 0$?

12. Show that the function given by $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is not continuous at $x = 0$.

13. Find $\frac{dy}{dx}$ at (4,9) if $\sqrt{x} + \sqrt{y} = 5$

14. If $y = \tan^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

15. If $xy = 9$, find $\frac{dy}{dx}$.

16. If $\frac{x}{a} + \frac{y}{b} = 1$, find $\frac{dy}{dx}$.

17. If $y = e^{-3\log x}$, then find $\frac{dy}{dx}$.

18. Differentiate $y = e^x \log(\sin 2x)$ with respect to x.

19. If $y = e^x + e^{-x}$, prove that $y'' = y$.

20. Verify the Rolle's Theorem for the function $f(x) = \sin 2x$ in $[0, \pi]$.

4- MARK QUESTIONS :

* Find the value of k so that the function defined by

21. $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x, & x \geq \pi \end{cases}$ is continuous at $x = \pi$.

(foreign 2011)

22. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}}\right)$.

23. Differentiate $\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)$ with respect to *x*.

24. Differentiate the function $\left(x^2+y^2\right)^2 = xy$ with respect to *x*.

25. If $x^{16}y^9 = \left(x^2+y\right)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$ (foreign 2012)

26. Find the derivative of $\tan^{-1}\left(\sqrt{1+x^2}-x\right)$

27. Verify the Rolle's Theorem for the function $f(x) = (x-1)(x-2)^2$ in [1,2]

28. Find $\frac{dy}{dx}$, if $y = (\log x)^x + x^{\log x}$ (Delhi 2013)

29. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

30. Verify Mean Value Theorem for the function $f(x) = x^2 + 2x + 3$ in [4,6]

6 - MARK QUESTIONS

31. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & if \quad x \le \frac{\pi}{2} \\ a, & if \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & if \quad x \ge \frac{\pi}{2} \end{cases}$. If $f(x)$ be continuous at $x = \frac{\pi}{2}$, find a and b. (Delhi 2016)

32. Show that the function $f(x) = |x-3|$, $x \in R$ is continuous but not differentiable at $x = 3$. (Delhi 2013)

33. Examine the continuity of the function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & x \ne 4 \\ 0, & x = 4 \end{cases}$ at $x = 4$.

34. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$.

35. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ (AI 2013)

36. Differentiate $x^{x\cos x}+\frac{x^2+1}{x^2-1}$ with respect to *x*. (Delhi 2011)

37. If $x=\sin t$ and $y=\sin pt$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0.$

38. If $x^py^q=(x+y)^{p+q}$, prove that (i) $\frac{dy}{dx}=\frac{y}{x}$ and (ii) $\frac{d^2y}{dx^2}=0$. (foreign 2012, 14)

39. If $y=\sqrt{\sin x+\sqrt{\sin x+.....}}$, prove that $\frac{dy}{dx}=\frac{\cos x}{2y-1}$.

40. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for -1 < *x* < 1, show that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$ (foreign 2012)

APPLICATION OF DERIVATIVES:
1 MARK QUESTIONS

1. Find the slope of the tangent to the curve $y=3x^2-6$ at the point on it whose x-coordinate is 2.
(Delhi 2009)
2. The total cost C(x) associated with provision of free mid-day meals to x students of a school in primary classes is given by $C(x)=0.005x^3-0.02x^2+30x+50$. If marginal cost is given by rate of change $\frac{dC}{dx}$ of the total cost, find the marginal cost of food for 300 students.
(Delhi 2013)
3. The radius of a circle is increasing at the rate of 0.7 cm/sec. what is the rate of increase of its circumference?
4. Find the point on the curve $y^2=8x$ for which the abscissa and ordinate change at the same rate.
5. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then find the radius.
6. Show that the function $y=4x-9$ is increasing for all $x\in R$.
7. At what point on the curve $x^2+y=4$, tangent is parallel to the X-axis?
8. It is given that at x=1, the function $f(x)=x^4-62x^2+ax+9$ attains its maximum value on the interval [0,2]. Find the value of *a*.
9. Find the maximum and minimum values if any of the function given by $f(x)=-(x-1)^2+10.$
10. Show that the function $y=\frac{3}{x}+7$ is strictly decreasing for $x\in R$

$(x \neq 0)$.

2 MARK QUESTIONS :

11. A circular disc of radius 3cm is being heated. Due to expansion, its radius increases at the rate of 0.05cm/s. Find the rate at which its area is increasing when radius is 3.2cm.
12. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.
13. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which tangent is parallel to X- axis.
14. For what values of x is the rate of increase of $x^3 - 5x^2 + 5x + 8$ twice the rate of increase of x ?
15. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.
16. Show that the function given by $f(x) = \sin x$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
17. At what point on the curve $y = x^2$ does the tangent make an angle of $45°$ with the X- axis?
18. If $y = x^2 + 5$ and x changes from 2 to 2.3, then find the approximate change in y.
19. Find the maximum and minimum values, if any, of the function given by $g(x) = x^3 + 1$
20. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which y-coordinate is changing twice as fast as x-coordinate.

4- MARK QUESTIONS :

21. A stone is dropped into a quiet lake and waves move in circles at a speed of 5cm/s. At the instant when the radius of the circular wave is 8cm, how fast is the enclosed area increasing?
22. A spherical balloon is being inflated by pumping in 16cm^3/s of gas. At the instant when balloon contains 36π cm^3 of gas, how fast is its radius increasing?
23. Find the intervals in which the function given by $f(x) = x^3 - 12x^2 + 36x + 17$ is increasing or decreasing.

24. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.
(foreign 2012)

25. Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$, where tangent is parallel to the y- axis.

26. Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$, at $t = \frac{\pi}{4}$.

27. Using differential, find the approximate value of $\sqrt{49.5}$
(Delhi 2012)

28. Find the absolute maximum value and the absolute minimum value for the function $f(x) = 4x - \frac{1}{2}x^2$, in the given interval $x \in \left[-2, \frac{9}{2}\right]$

29. Using differential, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$

30. Find the local maxima and local minima, if any, of the function given by, $f(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$

6 - MARK QUESTIONS :

31. Sand is pouring from a pipe at the rate of $12 cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one- sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?
(AI 2012)

32. Find the values of x for which the function $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to the X- axis. (Delhi 2010)

33. Find the value of p for which the curves $x^2 = 9p(9-y)$ and $x^2 = p(y+1)$ cut each other at right angles.
(AI 2015)

34. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be the least when the depth of the tank is half of its width.

35. At what points will the tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to the X- axis? Also find the equations of the tangents to the curve at these points.

36. Of all the rectangles each of which has perimeter 40m, find one

which has maximum area. Find the area also.

37. Manufacturer can sell x items at a price of Rs$\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs$\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit.

38. A wire of length 36cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum.

39. Show that the right circular cylinder of given volume and open at the top, has minimum total surface area , provided its height is equal to radius of its base. (foreign 2014)

40. Find two positive numbers whose sum is 16and sum of whose cubes is minimum.

Integrals

1 Mark Questions

1*. $\int \frac{\cos^9 x}{\sin x}\,dx$

2. $\int \frac{1}{\sqrt{9-25x^2}}\,dx$

3. $\int \frac{1}{x^2+2x+5}\,dx$

4. $\int \frac{1}{1+x-x^2}\,dx$

5. $\int \frac{1}{\sqrt{x^2+4x+2}}\,dx$

6. $\int \frac{1}{\sqrt{1+x-x^2}}\,dx$

7*. $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$

8. $\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$

4 Marks Questions

9. $\int \frac{2x+5}{x^2-x-2}\,dx$

10*. $\int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

11. $\int \frac{3x+1}{\sqrt{5-2x-x^2}}\,dx$

12. $\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$

13*. $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$

14*. $\int \frac{x}{(x-1)(x^2+4)}\,dx$

15*. $\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}\,dx$

16. $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$

17. $\int \frac{x^2+x-1}{x^2+x-6}\,dx$

18. $\int \frac{\sin 2x}{\sqrt{\cos^4 x-\sin^2 x+2}}\,dx$

19. $\int \frac{e^x}{e^{2x}+6e^x+5}\,dx$

20*. $\int \frac{1}{x(x^4+1)}\,dx$

21. $\int x\tan^{-1} x\,dx$

22. $\int \sec^3 x\,dx$

23*. $\int x^3 \log 2x\,dx$

24. $\int \frac{x^2+1}{x^4+1}dx$

25*. $\int \frac{1}{\sin^4 x+\cos^4 x}dx$

26.* $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

27. $\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx$

28. $\int \frac{\sin 2x+3\cos x}{(\sin^2 x+3\sin x+5)^4}dx$

29. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$

30. $\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}\,dx$

31. $\int \frac{\cos x}{\cos 3x}dx$

32. $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx$

33. $\int \frac{\sin x}{\sin 4x}dx$

34. $\int (x+1)\sqrt{1-x-x^2}\,dx$

35. $\int \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx$

36. $\int \frac{\sin 2x+3\cos x}{(\sin^2 x+3\sin x+5)^4}dx$

APPLICATION OF INTEGRALS
6 Marks

1. Using integration, find the area of the region $\{(x,y): x^2+y^2 \le 2ax, y^2 > ax, x, y \ge 0\}$.
2. Using integration, find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2+y^2=4$ at $(1,\sqrt{3})$.
3. Sketch the region bounded by the curves $y=\sqrt{5-x^2}$ and y =

$|x-1|$ find its area using ntegration.

4. Using integration, find the area of the region bounded by the curves:$y = |x+1| + 1, x = -3$
5. Prove that the curves
   $y^2 = 4x \; and \; x^2 = 4y$ divide the area of the square bounded by $x = 0,$
   $x = 4, y = 4 \; and \; y = 0$ into three equal parts.
6. Using integration, find the area bounded by the tangent to the curve $4y = x^2$
   $at \; the \; point \; (2,1) \; a$nd the lines whose equations are $x = 2y \; and \; x = 3y - 3$
   .
7. Find the area of the region enclosed between the two circles
   $x^2 + y^2 = 1 \; and \; (x-1)^2 + y^2 = 1.$
8. Find the area of the region $\{(x,y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ using method of integration.
9. Using integration, find the area of the triangle ABC, where A is (2,3),B is (4,7) and C is (6,2).
10. Draw the graph of
    $y = |x+1|$ and using integration find the area below $y = |x+1| \; above \; x -$ axis and between $x = -4 \; to \; x = 2.$

DIFFERENTIAL EQUATIONS

1mark

1. Write the order and degree of the differential equation
   $xy\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
2. If $\cos x.\frac{dy}{dx} + ysinx = tan^2x$ is a differential equation, then find its order
   and degree.
3. Write the differential equation formed from the equation y = m x + c,
   where m and c are arbitrary constants.
4. Write the differential equation obtained by eliminating the arbitrary constant
   C in the equation representing the family of curves $xy = C\cos x.$
5. If m and n are the order and degree, respectively of the differential equation
   $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x,$ then write the value of m + n.
6. Find the differential equation of the family of lines passing through the
   origin.

7. Find the solution of the differential equation $\frac{dy}{dx} = x^3e^{-2y}$.

8. Write the integrating factor of the differential equations $\sqrt{x}\ \frac{dy}{dx} + y = e^{-2\sqrt{x}}$.

9. Solve the differential equation $\frac{dy}{dx} = e^{x-y} + x^3e^{-y}$

10. Find the differential equation representing the family of curves $V = \frac{A}{r} + B$, where A and B are arbitrary constants.

2marks

1. Show that the solution of the differential equation $y = (2x^2 - 1) + ce^{-x^2} is\ \frac{dy}{dx} + 2xy - 4x^3 = 0$
2. Form the differential equation of the equation $y = a\cos 2x + b\sin 2x$, where a and b are constant. Obtain the differential equation of the family of circles passing through the points (a, 0) and (-a, 0).
3. Solve the differential equation $\frac{dy}{dx} = \sqrt{\frac{1-\cos x}{1+\cos x}}$.
4. Find the general solution of the differential equation $\frac{dy}{dx} = e^{3x-4y}$
5. Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = x + 1$
6. Find the general solution of differential equation $\frac{dy}{dx} + y = e^{-x}$.
7. Find the sum of the order and degree of the following differential equation

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

8. Solve the differential equation $xdy = -(y - x^3)dx$
9. Solve the differential equation $e^x \tan y dx + (1 - e^x)\, sec^2 y dy = 0$
10. Find the differential equation of all lines in XY –plane.

6marks

1. Form the differential equation of the family of Circles in the second quadrant and touching the coordinate axes..
2. If $\cos(a + y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$. Hence sow that $sina\frac{d^2y}{dx^2} + \sin 2(a + y)\frac{dy}{dx} = 0$
3. Solve the following differential equation $(1 + y^2)dx = (\tan^{-1} y - x)dy$.
4. Find the particular solution of the differential equation

$$(1 + x^2)\frac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right) \text{ given that } y = 1 \text{when } x = 0$$

5. Solve the differential equation $x^2dy + (xy + y^2)dx = 0$ given $y = 1$, when $x = 1$.
6. Solve the following differential equation

$$x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx)$$

7. Show that the differential equation $\left(xe^{\frac{y}{x}} + y\right)dx = xdy$ is homogeneous.Find the particular solution of this differential equation, given that x=1 when y=1.
8. Find the general solution of the differential equation $(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x\backslash$
9. Find the particular solution of the differential equation :

$$xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}sin\left(\frac{y}{x}\right) = 0. for\ x = 1, y = 0$$

10. Show that the differential equation $\left[xsin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$ is homogeneous. Find the particular solution of the differential equation, given that $y = \frac{\pi}{4}$ when $x = 1$.

3D GEOMETRY

1 Mark

1) Find the Cartesian equation of the line which passes through the point (-2,4,5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = z = 8$
2) Find the vector equation of a plane at which is at a distance of 7 units from the origin and the normal to the vector $(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k})$.
3) Find the intercepts cut off by the plane 2x +y-z+5=0
4) Find the distance of a point (2,5-3) from the plane $\vec{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\right) = 4$
5) Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

2marks

6) Find the vector and the Cartesian equation of the line that passes through the points (,3,-2,-5), (3,-2,6)
7) Find the angle between the pair of the lines given by $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} and\ \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$
8) Find the angle between the two planes 2x + y +3z-2=0 and x -

2y +5 =0

9) Find the Cartesian equation of the line which passes through the point (2,-1,-1) and parallel to the line given by $6x - 2 = 3y + 1 = 2z - 2$

10) Find the vector and Cartesian equations of the plane that passes through the point (0,1,-2)and having normal vector $\hat{\imath} + \hat{\jmath} - \hat{k}$.

4marks

11) Find the shortest distance and the vector equation of the line of shortest distance between the lines given by

$$\boldsymbol{r} = 3\boldsymbol{i} - 15\boldsymbol{j} + 9\boldsymbol{k} + \lambda(2\boldsymbol{i} - 7\boldsymbol{j} + 5\boldsymbol{k}) and\ \boldsymbol{r} = (2\mu - 1)\boldsymbol{i} + (1 + \mu)\boldsymbol{j} + (9 - 3\mu)\boldsymbol{k}$$

12) Find the distance of the point (1,-2,3) from the plane x-y+z=5, measured along a line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

13) Find the foot of the perpendicular from the point (1,3,4) to the plane

2x –y+z+3=0 also find the image point.

14) Find the valves of p so that the line

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} and\ \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}\ are\ right\ angles$

15) Find the equation of the plane which contains the line of intersection of the planes$\vec{r}.(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0: \vec{r}.(2\hat{\imath} + \hat{\jmath} + \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$:

VECTOR ALGEBRA

1 marks

1) Find the magnitude of the following vector:- $\vec{a} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$

2) Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$.

3) Find the Vector Joining the points P(5,3,0) and Q(-1,-2,-4) Q to P.

4) Find the position vector of the midpoint of the vector joining the points

P(,3-2,0) and Q(1,-1,2).

5) Find the projection of the vector $\vec{a}$=$2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

2 Marks

6) If a is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8, then\ find\ |\vec{x}|$

7) Find the area of a triangle having the points A(1,2,3), B(2-1,1)

and C(-1,2,3,) as its vertices.

8) Find the area of a parallelogram whose adjacent sides are determined by the vectors **a**=$\hat{\imath} - \hat{\jmath} + 3\hat{k}$ *and* $\boldsymbol{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

9) Given $|\vec{a}| = 13, \left|\vec{b}\right| = 5, and\ \vec{a}.\vec{b} = 60\ .find\ \left|\vec{a}\ x\vec{b}\right|$.

10) Find $\lambda\ and\ \mu$ if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k})\ x\ (\hat{\imath} + \lambda j + \mu k\ ) = \vec{0}$

4 Marks

11) If $\vec{a}$ and $\vec{b}$ are Unit vectors Inclined at an angleθ, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|$.

12) If with reference to the right handed system of mutually perpendicular unit vectors $\hat{\imath}, \hat{\jmath}\ and\ \hat{k}, \vec{\alpha} = 3\hat{\imath} - \vec{\jmath}\ \ ,\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then express $\overrightarrow{\beta}$ in the form $\vec{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$.

13) $\vec{a}$ = $\hat{\imath}$ + 4$\hat{\jmath}$ +2$\hat{k}$, $\vec{b}$ =3 $\hat{\imath}$ - 2$\hat{\jmath}$ +7$\hat{k}$, $\vec{c}$ = 2 $\hat{\imath}$ - $\hat{\jmath}$ +4$\hat{k}$,Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ & $\vec{c}.\vec{d}$ = 15

14) If $\vec{a}, \vec{b}$ are two vectors then prove that $(1 + |\vec{a}|^2)\left(1 + \left|\vec{b}\right|^2\right) = \left\{1 - \vec{a}.\vec{b}\right\}^2 + \left[\vec{a} + \vec{b} + \left(\vec{a}\ x\ \vec{b}\right)\right]^2$

15) Show that the angle between two diagonals of a cube is $\cos^{-1}1/\sqrt{3}$

LINEAR PROGRAMMING PROBLEMS

2Marks

1) A company sells two different products A and B . The two products are produced in a common production process which has a total capacity of 500 man hours . It takes 5 hours to produce a units of A and 3 Hours to produce a unit B , the demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125 . Profit on each unit of A is Rs20 and that on B is Rs 15.Form the constrains to solve this problem.
2) A manufacturing Company makes two models A and B of a product. Each piece of model A requires 9 labor hours for fabricating and 1 lab our hour for finishing . Each piece of Model

B requires 12 lab our hours of fabricating and 3 hours for finishing . For fabricating and finishing , the maximum lab our hours available are 180 and 30 respectively. The company makes a profit of Rs8000 on each piece of model A and Rs12000 on each piece of model B. Form the constrains to solve this problem.

3) A manufacture has three machines I,II, and III installed in his factory . Machines I and II are capable of being operated for at most 12 hours where as machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing I unit of each of M and N on the three machines are given in the following table.

Items	Numbers of hours required on the machine		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs600 and Rs400 on items M and N respectively .Form constrains to find how many of each items should she produce so as to maximize her profit assuming that she can sell all the items that she produced?

4Marks

4) Solve the following linear programming problem graphically: Maximize Z=60X +15Y subject to constraints $x + y \leq 50, y \leq 90 \; and \; x \geq 0. y \geq 0$

5) Solve the following linear programming problem graphically: Minimize Z=X - 15Y+20 subject to constraints $x - y \geq 0, -x + 2y \geq 2 \; and \; x \geq 3. y \leq 4. . x \geq 0. y \geq 0.$

6Marks

6) A oil company requires13000,20,000,and 15,000 barrels of high grade, medium and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high grade , medium and low grade oil respectively, where as refinery B produces 200, 400 and 100 barrels per day respectively . If A Cost Rs400 per day and B cost rs300 per day to operate, how Many days should each refinery be run to minimize the cost, meeting the requirements (Ans: 170/3 , 110/3)

7) Two go downs A and B have gain capacity of 100 quintals and 50 quintals respectively . They supply to 3 ration shops, D, E and F whose requirements are 60,50 and 40 quintals respectively . The cost of transportation per quintal from the go downs to the shops

are given in the following table:

Transportation cost per quintal (In Rs)		
From/To	A	B
D	6	4
E	3	2
F	2.50	3

How should be the supply be transported in order the transportation cost is minimum? What is the minimum cost?

8) An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of Rs. 400 is made on each economy ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the Air line fund. The Air line reserves at least 20 seats for executive class. However at least 4times as many passengers prefer to travel by Economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the net profit of the Airline. Make the above as an LPP and solve graphically. Do you think more passengers would prefer to travel by such an Air line than by others.

9) If a young man rides his motor cycle at 25km/h, he had to spend Rs2per km on petrol. If he rides at a faster speed of 40km/h , the petrol cost increases Rs 5 per km.He has Rs100 to spend on petrol and wishes to find what is the maximum distances he can travel within one hour .Express this as on L.P.P and solve it graphically.

10) Beena wishes to mix two types of food P and Q in such a way that the vitamin content of the mixture contains at least units of vitamin A and 11 units of vitamin B . food P costs Rs60 kg and food Q cost 80 Kg. Food P contains 3 units /kg of vitamin A and 5 unit /kg of vitamin B while food Q contains 4units /kg of vitamin A and 2 units/kg of vitamin B . Determine the minimum cost of the mixture.

11) A dietician wishes to mix altogether two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 Units of vitamin C. The vitamin content of one kg food is given below

Food	Vitamin A	Vitamin B	Vitamin C

X	1	2	3
Y	2	2	1

One kg of food cost Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet.

PROBABILITY:

1 MARK QUESTIONS :

1. Given P(A) = 0.2, P(B) = 0.3 and $P(A \cap B)$ = 0.1. Find P(A/B).
2. Given P(A) = 0.4, P(B) = 0.7 and P(B/A) = 0.6. Find $P(A \cup B)$.
3. Given $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A \cap B)=\frac{1}{6}$. Are the events A and B independent?

2 MARK QUESTIONS :

4.. 12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

5. Given that two numbers appearing on throwing two dice are different. Find the probability of the event ' the sum of numbers on the dice is 4'.

4- MARK QUESTIONS :

6. A family has 2 children. Find the probability that both are boys, if it is known that (i) atleast one of the children is a boy (ii) the elder child is a boy. (AI 2010)
7. Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls, given that (i) the youngest is a girl (ii) at least one is a girl. (Delhi 2014)
8. A husband and a wife appear in an interview for two vacancies for the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that (i) both will be selected? (ii) only one of them will be selected? (iii) none will be selected?
9. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of the machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective. (foreign 2015)
10. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs.

(Delhi 2010)

11. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3, what is the probability that the student knows the answer, given that he answered it correctly? (foreign 2010)
12. Two cards are drawn simultaneously (or successively without replacement) from a well- shuffled pack of 52 cards. Find the mean and variance of the number of red cards. (AI 2012)
13. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) P(X< 3) (iii) P(X> 6) (iv) P(0 < X < 3) (AI 2011)

14. How many times must a man toss a fair coin so that the probability of having at least one head is more than 80% ? (Delhi 2012)
15. Five cards are drawn one by one with replacement from a well-shuffled deck of 52 cards. Find the probability that (i) all the five cards are diamonds (ii) only 3 cards are diamonds (iii) none is diamond. (foreign 2014)