PRACTICE QUESTIONS FOR COMPETITIVE EXAMINATIONS

SUBJECT: MATHEMATICS

TOPIC: TANGENTS AND NORMALS

1. Let $f(x) = x^3 + ax + b$ with $a \neq b$ and suppose the tangent lines to the graph of f at x = a and x = b have the same gradient. Then the value of $f(1)$ is equal to -

(A) 0 (B) 1 (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

2. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0,1) and also touches the x-axis at the point (–1,0) then the values of x for which the curve has a negative gradient are -

(A) $x > -1$ (B) $x < 1$ (C) $x < -1$ (D) $-1 \le x \le 1$

3. The lines tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^4 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

4. A 13 ft. ladder is leaning against a wall when its base starts to slide away. At the instant when the base is 12 ft. away from the wall, the base is moving away from the wall at the rate of 5 ft/sec. The rate at which the angle θ between the ladder and the ground is changing is -

(A) $-\frac{12}{13}$ rad / sec. (B) –1 rad/sec. (C) $-\frac{13}{12}$ rad / sec. (D) $-\frac{10}{13}$ rad / sec.

5. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point-

(1) (a, 0) (2) (0, a) (3) (0, 0) (4) (a, a)

6. The angle of intersection of $x = \sqrt{y}$ and $x^3 + 6y = 7$ at (1, 1) is -

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

7. Which of the following pair(s) of curves is/are orthogonal.

(A) $y^2 = 4ax$; $y = e^{-x/2a}$ (B) $y^2 = 4ax$; $x^2 = 4ay$

(C) $xy = a^2$; $x^2 - y^2 = b^2$ (D) $y = ax$; $x^2 + y^2 = c^2$

8. The coordinates of the point(s) on the graph of the function, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$ where the tangent drawn cut off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, is -

(A) (2,8/3) (B) (3,7/2) (C) (1,5/6) (D) none

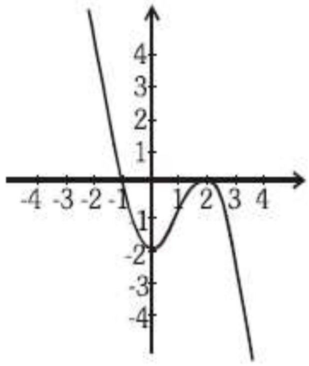
9. Equation of common tangent(s) of $x^2 - y^2 = 12$ and $xy = 8$ is (are) -

(A) $y = 3x + 4\sqrt{6}$ (B) $y = -3x + 4\sqrt{6}$ (C) $3y = x + 4\sqrt{6}$ (D) $y = -3x - 4\sqrt{6}$

10. The angle at which the curve $y = Ke^{Kx}$ intersects the y-axis is -

(A) $\tan^{-1}k^2$ (B) $\cot^{-1}(k^2)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$ (D) $\sec^{-1}\left(\sqrt{1+k^4}\right)$

11. A cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ has a graph which is tangent to the x-axis at 2, has another x-intercept at −1, and has y-intercept at −2 as shown. The values of, a + b + c + d equals-



(A) −2

(B) −1

(C) 0

(D) 1

12. Consider the curve represented parametrically by the equation

$$x = t^3 - 4t^2 - 3t \text{ and } y = 2t^2 + 3t - 5 \text{ where } t \in R$$

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then-

(A) H = 2 and V = 1 (B) H = 1 and V = 2 (C) H = 2 and V = 2 (D) H = 1 and V = 1

13. Let $f(x)$ be a nonzero function whose all successive derivative exist and are nonzero. If $f(x)$, $f'(x)$ and $f''(x)$ are in G.P. and $f(0) = 1$, $f'(0) = 1$, then -

(A) $f'(x) < 0 \ \forall x \in R$ (B) $f''(x) < 1 \ \forall x \in R$

(C) $f''(0) \neq f'''(0)$ (D) $f''(x) > 0 \ \forall x \in R$

ASSERTION & REASON

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

14. Statement-I : The product of the ordinates to the point of tangency to the curve $(1 + x^2)y = 2 - x$, where the tangent makes equal intercept with coordinate axes is equal to 1.

Because

Statement-II : Slope of straight line making equal intercept with coordinate axis is equal to 1.

(A) A (B) B (C) C (D) D

15. Statement-I : Any tangent to the curve $y = x^7 + 8x^3 + 2x + 1$ makes an acute angle with the positive x-axis.

Because

Statement-II : Any tangent to the curve $y = a_0x^{2n+1} + a_1x^{2n-1} + a_2x^{2n-3} + \ldots\ldots + a_nx + 1$ makes an acute angle with the positive x-axis where $a_1, \ldots\ldots, a_{n-1} \geq 0$; $a_0, a_n > 0$ and $n \in N$.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Consider the function $f(x) = x^2 f(1) - xf'(2) + f''(3)$ such that $f(0) = 2$

On the basis of above information, answer the following questions :

The values of $f'(1)$ is -

16. (A) 0 (B) 1 (C) 2 (D) 1

Equation of tangent to $y = f(x)$ at $x = 3$ is -

17 (A) $y = x - 7$ (B) $y = \frac{x}{4} - 7$ (C) $y = 4x - 7$ (D) none of these

The angle of intersection of $y = f(x)$ and $y = 2e^{2x}$ is -

18 (A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) 0 (D) $\tan^{-1}\left(\frac{6}{7}\right)$

19. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\, dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 4

20. If $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$, for all $x_1, x_2 \in R$. Find the equation of tangent to the curve $y = f(x)$ at the point (1, 2).

A) $y - 2 = 0$ B) $x - 2 = 0$ C) $x - y = 0$ D) $x + 2 = 0$

ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	C	D	B	1	D	ACD	AB	BD	BC	B	B	D	C	A	A	C	D	1	A